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**Research**

# Implementation of Nonstandard Finite Difference Method for Autonomous First Order Ordinary Differential Equations

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## Abstract

This paper presents nonstandard finite difference methods for the solution of autonomous first order ordinary differential equations. The scheme is shown to be convergent, stable and preserve some essential properties of the exact solution.

**Keywords:** Nonstandard, finite difference, autonomous, nonlocal approximation, stability, monotone dependence on initial value.

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## Introduction

Most problems arising from the physical, biological, social sciences, management sciences and engineering are modeled in differential equations; it has been established that most of these models cannot be easily solved by analytical means. Therefore, approximate solutions are needed which are generated by numerical techniques called Standard Methods.

Mickens (1994, 2000), Anguelov and Lubuma (2000) and Ibijola et al (2008) established that standard methods have some shortcomings of not transferring some qualitative properties of the exact solution to the numerical solution. In practice, the limit of the step-size used is not reached and what is obtained is the numerical solution of one or several values of the step-size. These shortcomings may create a lot of problems which may affect the stability of the standard approach. Ibijola et al. (2008). These brought the need for nonstandard finite difference schemes introduced by Mickens (1994).

**1.1 Standard Finite Difference Modeling Rules**

We shall begin by considering the initial value problem for an autonomous first-order ordinary differential equation of the form

$$\frac{dy}{dt} = f(y),\ y(t_0) = y_0 \qquad (1)$$

Where f(y) is in general a nonlinear function of y. We shall also assume that equation (1) satisfies the popular Lipshitz condition.

For the numerical approximation of equation (1) we shall have a uniform lattice with step – size $\Delta t = h$ we can replace the independent variable t by

$$t \to t_n = nh \qquad (2)$$

Where n is an integer i.e. $t \in \{..., -2, -1, 0, 1, 2, 3, ...\}$ (3)

The independent variable y(t) is replaced by

$$y(t) \to y_n \qquad (4)$$

Where $y_n$ is the approximation of $y(t_n)$. Likewise the function $f(y)$ is replaced by

$$f(y) \to f_n \qquad (5)$$

Using equation (1) we can replace $\frac{dy}{dt}$ by any of the following forms;

$$\frac{dy}{dt} \to \begin{cases} \frac{y_{n+1} - y_n}{h} \\ \frac{y_n - y_{n-1}}{h} \\ \frac{y_{n+1} - y_{n-1}}{2h} \end{cases} \qquad (6)$$

These representations of discrete first derivative are known respectively as forward Euler, backward Euler, Central difference schemes.

Thus, a simple finite-difference method using forward Euler is given by;

$\frac{y_{n+1} - y_n}{h} = f(y_n)$ which gives

$$y_{n+1} = y_n + hf(y_n) \qquad (7)$$

**1.2 Non - Standard Finite Difference Rules**

The general form of non-standard method can be written as

$$y_{n+1} = F(h, y_n) \qquad (8)$$

Mickens(1994,2000), (Ibijola, E.A,.Omowaye, A.J. and O.A.Ade-Ibijola,2009)

Nonstandard finite difference schemes were developed using a collection of rules set by Mickens. In particular nonstandard finite difference schemes were defined by using two of Mickens rules: We shall go further by stating some definitions and theorems.

**Definition 1**Mickens (1994)

The scheme (8) is called a nonstandard finite difference method if at least one of the following condition is met;

(i) In the first-order discrete derivative that occurs in (8), the traditional denominator h is replaced by a non-negative function, $\phi(h)$ such that $\phi(h) = h + O(h^2)$;

as $h \to 0$ (9)

Where the denominator function $\emptyset(h)$ can be chosen to be any of the following functions

$$\phi(h) = \begin{cases} h, \\ \sin(h), \\ e^h - 1, \\ 1 - e^h, \\ \frac{1-e^{-\lambda h}}{\lambda}, \\ etc. \end{cases} \quad (10)$$

(ii) Nonlinear terms of $f(y)$ are modeled nonlocally by approximating them at different grid points, e.g. $y^2 \to y_n y_{n+1}$

**Definition 2** Anguelov, R. and Lubuma J.M.S. (2003)

Assume that equation (1) satisfy some property $P$. The numerical scheme (8) is called (qualitative) stable with respect to property $P$ (or $P$ - stable). If for every value of h>0 the set of solutions of (8) satisfies property $P$.

We shall assume that function $F(h, y_n)$ in (8) has continuous derivatives with respect to both variable for $h > 0, y \in R$ and that

$$F(0, y) = y \text{ and } \frac{\partial F}{\partial h}(0, y) = f(y) \quad (11)$$

At this point it is necessary to note that consistency implies (11) if y is the solution of the differential equation. The following theoremswill be of good use.

**Theorem 1 (** Anguelov, R. and Lubuma J.M.S. 2000)

The difference scheme (8) is stable with respect to monotone dependence on initial value if

$$\frac{\partial F}{\partial h}(h, y) \geq 0,\ y \in R, h > 0 \quad (12)$$

**Definition 3**

A set $G(\Omega)$ of real – valued functions defined on a subset $\Omega$ of $[t_0, \infty)$ is said to monotonically depend on the initial value at $t_0$.

If for every two function $y, z \in G(\Omega)$ we have

$$y(t_0) \leq z(t_0) \Rightarrow y(t) \leq z(t),\ t \in \Omega \quad (13)$$

It is necessary to note that since equation (1) is assumed to satisfy Lipshitz Condition, the set of solution for equation (1) is monotonically dependent on the initial value at $t_0$.

**Definition 4**

The finite difference scheme (8) is stable with respect to monotonocity of solutions if for every $y \in R$, the solution $y_n$ of (8) is an increasing or decreasing sequence.

**Theorem 2**

Assume that the finite difference scheme (8) is stable with respect to monotone dependence on initial value.

Assume also that for every $h > 0$, the equations

$$y = F(h, y) \text{ and } f(y) = 0 \quad (14)$$

have the same roots considered with their multiplicity, then the finite difference scheme (8) is stable with respect to monotonocity of solutions. If the condition in (14) is satisfied then the difference scheme (8) is elementary stable.

## 2. Main results

### 2.1 Non-standard finite difference schemes

The nonlinear term in the right side of equation (1) can be approximated nonlocally in many different ways, but in this paper we will approximate this initial value problem:

$$y' = 1 + y^2 \tag{15}$$

Using the following approximations

$$1 + y^2 \cong 1 + ay_n^2 + (1-a)y_n y_{n+1},$$

$$a \in R \tag{16}$$

$$1 + y^2 \cong 1 + y_n y_{n+1}. \tag{17}$$

$$1 + y^2 \cong 1 + y_n \frac{y_{n-1} + y_{n+1}}{2} \tag{18}$$

Considering the nonstandard finite difference scheme in equation (16) we have a Nonstandard finite difference scheme as follows:

$$\frac{y_{n+1} - y_n}{\phi(h)} = 1 + ay_n^2 + (1-a)y_n y_{n+1} \tag{19}$$

$$y_{n+1} - y_n = \phi(h) + ay_n^2\phi(h) + ((1-a)y_n y_{n+1})\phi(h)$$

Choosing like terms we have

$$y_{n+1} - ((1-a)y_n y_{n+1})\phi(h) = y_n + \phi(h) + ay_n^2\phi(h)$$

$$y_{n+1}(1 - ((1-a)y_n)\phi(h)) = y_n + \phi(h) + ay_n^2\phi(h)$$

This will result into the following scheme

$$y_{n+1} = \frac{y_n + \phi(h)(1 + ay_n^2)}{(1 - (1-a)y_n\phi(h)} \tag{20}$$

Using equation (17) we have the following nonstandard scheme

$$\frac{y_{n+1} - y_n}{\phi(h)} = 1 + y_n y_{n+1} \tag{21}$$

$$y_{n+1} - y_n = (1 + y_n y_{n+1})\phi(h)$$

$$y_{n+1} - y_n y_{n+1}\phi(h) = y_n + \phi(h)$$

$$y_{n+1}(1 - y_n\phi(h)) = y_n + \phi(h)$$

Then we have

$$y_{n+1} = \frac{y_n + \phi(h)}{1 - \phi(h)y_n} \tag{22}$$

Also using equation (18) we have the following nonstandard scheme

$$\frac{y_{n+1} - y_n}{\phi(h)} = 1 + y_n \frac{(y_{n-1} + y_{n+1})}{2} \tag{23}$$

$$2y_{n+1} - 2y_n = 2\phi(h) + y_n(y_{n-1} + y_{n+1})\phi(h)$$

$$2y_{n+1} - y_n y_{n+1}\phi(h) = 2y_n + 2\phi(h) + y_n y_{n-1}\phi(h)$$

$$y_{n+1} = \frac{2y_n + 2\phi(h) + y_n y_{n-1}\phi(h)}{(2 - y_n\phi(h))}$$

This will give us

$$y_{n+2} = y_{n+1}\frac{(2 + \phi(h)y_n) + 2\phi(h)}{2 - \phi(h)y_{n+1}} \tag{24}$$

This is a two – step numerical scheme that will need two starting points.

Let us also consider initial value problem $y' = -y^2$, and derive its non – standard scheme which we can approximate nonlocally as follows:

$$y(0) = 1 \tag{25}$$

$$y^2 \cong ay_n^2 + (1-a)y_n y_{n+1}, a \in R \tag{26}$$

$$y^2 \cong y_n y_{n+1}. \tag{27}$$

$$y^2 \cong y_n \frac{y_{n-1} + y_{n+1}}{2} \tag{28}$$

Considering equation (26) we have

$$\frac{y_{n+1} - y_n}{\phi(h)} = ay_n^2 + (1-a)y_n y_{n+1}$$

$$y_{n+1} - y_n = ay_n^2\phi(h) + (1-a)y_n y_{n+1}\phi(h)$$

Collecting like terms and simplifying we have following nonstandard method

$$y_{n+1} = \frac{y_n + ay_n{}^2\phi(h)}{1-(1-a)y_n\phi(h)} \tag{29}$$

Likewise considering equation (27) we have a nonstandard difference scheme

$$\frac{y_{n+1} - y_n}{\phi(h)} = y_n y_{n+1}$$

Collecting like terms and simplifying we have

$$y_{n+1} = \frac{y_n}{1 - y_n\phi(h)} \tag{30}$$

Considering equation (28) we have the following;

$$\frac{y_{n+1} - y_n}{\phi(h)} = y_n \frac{y_{n-1} + y_{n+1}}{2}$$

On simplifying as above we have the following non – standard scheme

$$y_{n+2} = \frac{2y_{n+1} + y_n y_{n+1}\phi(h)}{2 - y_{n+1}\phi(h)} \tag{31}$$

### 2.2 Qualitative stability properties of the schemes

Let us consider equation (20), it can generally be written as

$$F(h, y_n) = \frac{y_n + \phi(h)(1+ay_n^2)}{(1-(1-a)y_n\phi(h)} \tag{32}$$

With

$$F(h, y) = \frac{y_n + \phi(h)(1+ay_n^2)}{(1-(1-a)y_n\phi(h)} \tag{33}$$

We want to determine $\frac{\partial F(h,y)}{\partial y} > 0$ and we obtain the following;

$$\frac{\partial F(h,y)}{\partial y} = \frac{[1-(1-a)y\phi(h)]\frac{\partial}{\partial y}\left[y+(1+ay^2)\phi(h)\right] - \left[y+\phi(h)(1+ay^2)\right]\frac{\partial}{\partial y}[1-(1-a)y\phi(h)]}{[1-(1-a)y\phi(h)]^2}$$

Simplifying we obtain the following

$$\frac{\partial F(h,y)}{\partial y} = \frac{1 + ay\phi(h) + \phi(h)^2 - a\phi(h)^2}{[1-(1-a)y\phi(h)]^2} \quad (34)$$

Equation

(34) is true if $1 + ay\phi(h) + \phi(h)^2 - a\phi(h)^2 \geq 0$ (35)

For this to happen, $a < 0$.

We shall proceed to show that the schemes (22) and (24) are stable with respect to monotone dependence on initial value. Therefore,

$\frac{\partial F(h,y)}{\partial y} \geq 0$, $y \in R$, $h > 0$ when

$$F(h,y) = y_{n+1} = \frac{y + \phi(h)}{1 - \phi(h)y} \quad (35)$$

From equation (35) on simplification we have

$$\frac{\partial F(h,y)}{\partial y} = \frac{1 + \phi(h)^2}{[1-\phi(h)hy]^2} \quad (36)$$

Hence $\frac{\partial F}{\partial y} > 0$, since $\phi(h)$ is positive and $h > 0$. This implies that scheme (22) is stable with respect to monotone dependence on initial value.

Let us consider equation (24)

$$\frac{\partial F(h,y)}{\partial y} = y_{n+1} = \frac{y(2 + \phi(h)y) + 2\phi(h)}{2 - \phi(h)y}$$

This expression simplifies to the following

$$\frac{\partial F(h,y)}{\partial y} = \frac{4 - 3\phi(h)^2 y^2 + 2\phi(h)^2}{[2-\phi(h)y]^2} \geq 0 \quad (37)$$

The above equation holds if $y \leq 1$ and $h \leq 1$. Therefore scheme (24) will be stable with respect to monotone dependence on initial value and therefore elementary stable.

## 3. Implementation of the non-standard schemes

Using schemes (20), (22) and (24) to approximate initial value problem;

1. $y' = 1 + y^2,\ \ y(0) = 1$, in the interval $0 \leq x \leq \frac{\pi}{4}$, with theoretical solution given as $y(x) = \tan(x + \frac{\pi}{4})$ which is definitely unbounded at $x = \frac{\pi}{4}$.

**Table 1**

Step size h = 0.1

| s/n | $x_n$ | Scheme (20) | exact | Error (20) |
|---|---|---|---|---|
| 1 | 0.10 | 0.12227674E+01 | 0.12238381E+01 | 0.10707378E-02 |
| 2 | 0.20 | 0.15076777E+01 | 0.15095335E+01 | 0.18558502E-02 |
| 3 | 0.30 | 0.18938348E+01 | 0.18972181E+01 | 0.33832788E-02 |
| 4 | 0.40 | 0.24604676E+01 | 0.24672010E+01 | 0.67334175E-02 |
| 5 | 0.50 | 0.33966215E+01 | 0.34122157E+01 | 0.15594244E-01 |
| 6 | 0.60 | 0.52928143E+01 | 0.53411722E+01 | 0.48357964E-01 |
| 7 | 0.70 | 0.11404368E+02 | 0.11724983E+02 | 0.32061481E+00 |
| 8 | 0.80 | -0.50967169E+03 | -0.67028618E+02 | 0.44264307E+03 |
| 9 | 0.90 | -0.87198544E+01 | -0.86635237E+01 | 0.56330681E-01 |
| 10 | 1.00 | -0.45830908E+01 | -0.45810776E+01 | 0.20132065E-02 |

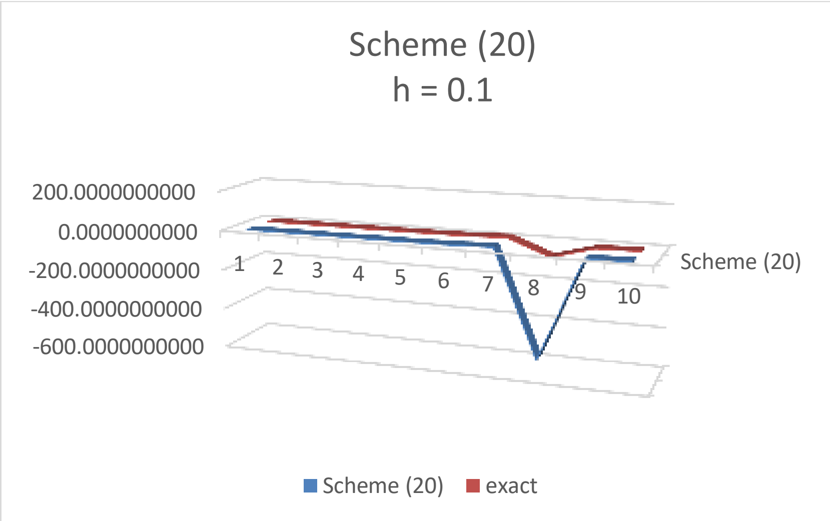


Figure 1. Graphical Representation of Scheme (20) on initial value problem 1 with step size h = 0.1

**Table 2**
Step size h = 0.01

| s/n | $x_n$ | Scheme (20) | exact | Error (20) |
|---|---|---|---|---|
| 0 | 0.01 | 0.10201817E+01 | 0.10208480E+01 | 0.66637993E-03 |
| 1 | 0.10 | 0.12227674E+01 | 0.12238381E+01 | 0.10707378E-02 |
| 2 | 0.20 | 0.15076777E+01 | 0.15095335E+01 | 0.18558502E-02 |
| 3 | 0.30 | 0.18938348E+01 | 0.18972181E+01 | 0.33832788E-02 |
| 4 | 0.40 | 0.24604676E+01 | 0.24672010E+01 | 0.67334175E-02 |
| 5 | 0.50 | 0.33966215E+01 | 0.34122157E+01 | 0.15594244E-01 |
| 6 | 0.60 | 0.52928143E+01 | 0.53411722E+01 | 0.48357964E-01 |
| 7 | 0.70 | 0.11404368E+02 | 0.11724983E+02 | 0.32061481E+00 |
| 8 | 0.80 | -0.50967169E+03 | -0.67028618E+02 | 0.44264307E+03 |
| 9 | 0.90 | -0.87198544E+01 | -0.86635237E+01 | 0.56330681E-01 |
| 10 | 1.00 | -0.45830908E+01 | -0.45810776E+01 | 0.20132065E-02 |

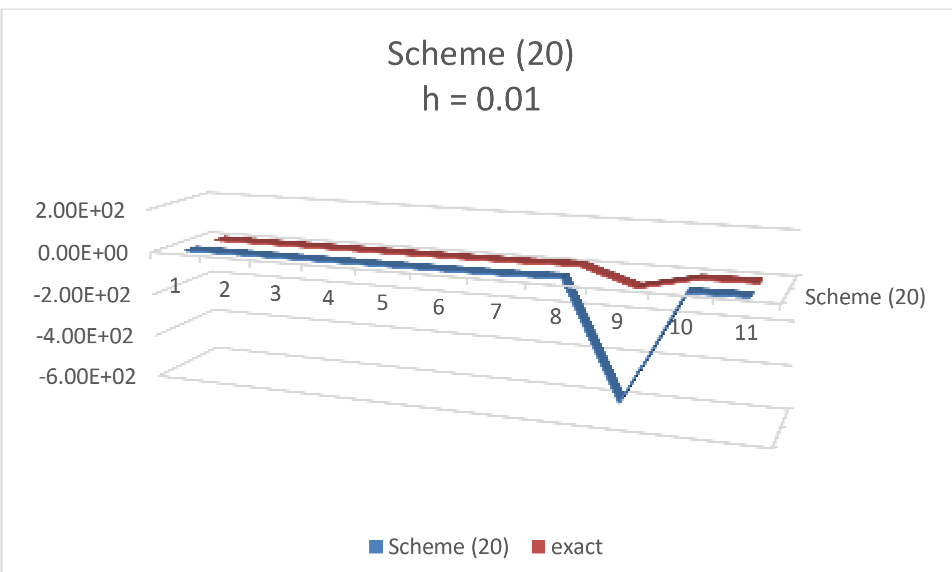


Figure 2. Graphical Representation of Scheme (20) on initial value problem 1 with step size h = 0.01.

**Table 3**
Step size h = 0.1

| s/n | $x_n$ | Scheme (22) | exact | Error (22) |
|---|---|---|---|---|
| 1 | 0.10 | 0.12230406E+01 | 0.12238381E+01 | 0.79751015E-03 |
| 2 | 0.20 | 0.15084760E+01 | 0.15095335E+01 | 0.10575056E-02 |
| 3 | 0.30 | 0.18957194E+01 | 0.18972181E+01 | 0.14986992E-02 |
| 4 | 0.40 | 0.24648685E+01 | 0.24672010E+01 | 0.23324490E-02 |
| 5 | 0.50 | 0.34080129E+01 | 0.34122157E+01 | 0.42028427E-02 |
| 6 | 0.60 | 0.53312674E+01 | 0.53411722E+01 | 0.99048615E-02 |
| 7 | 0.70 | 0.11678170E+02 | 0.11724983E+02 | 0.46813011E-01 |
| 8 | 0.80 | -0.68604897E+02 | -0.67028618E+02 | 0.15762787E+01 |
| 9 | 0.90 | -0.86899242E+01 | -0.86635237E+01 | 0.26400566E-01 |
| 10 | 1.00 | -0.45887733E+01 | -0.45810776E+01 | 0.76956749E-02 |

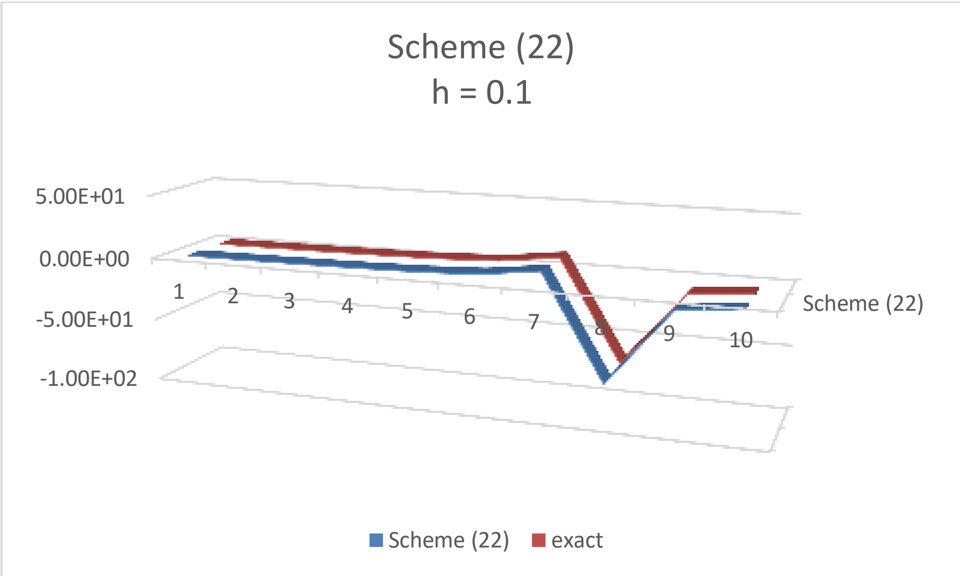


Figure 3. Graphical Representation of Scheme (22) on initial value problem 1 with step size h = 0.1.

**Table 4**
Step size h = 0.01

| s /n | $x_n$ | Scheme (22) | exact | Error (22) |
|---|---|---|---|---|
| 0 | 0.01 | 0.10202020E+01 | 0.10208480E+01 | 0.64599514E-03 |
| 1 | 0.10 | 0.12230406E+01 | 0.12238381E+01 | 0.79751015E-03 |
| 2 | 0.20 | 0.15084760E+01 | 0.15095335E+01 | 0.10575056E-02 |
| 3 | 0.30 | 0.18957194E+01 | 0.18972181E+01 | 0.14986992E-02 |
| 4 | 0.40 | 0.24648685E+01 | 0.24672010E+01 | 0.23324490E-02 |
| 5 | 0.50 | 0.34080129E+01 | 0.34122157E+01 | 0.42028427E-02 |
| 6 | 0.60 | 0.53312674E+01 | 0.53411722E+01 | 0.99048615E-02 |
| 7 | 0.70 | 0.11678170E+02 | 0.11724983E+02 | 0.46813011E-01 |
| 8 | 0.80 | -0.68604897E+02 | -0.67028618E+02 | 0.15762787E+01 |
| 9 | 0.90 | -0.86899242E+01 | -0.86635237E+01 | 0.26400566E-01 |
| 10 | 1.00 | -0.45887733E+01 | -0.45810776E+01 | 0.76956749E-02 |

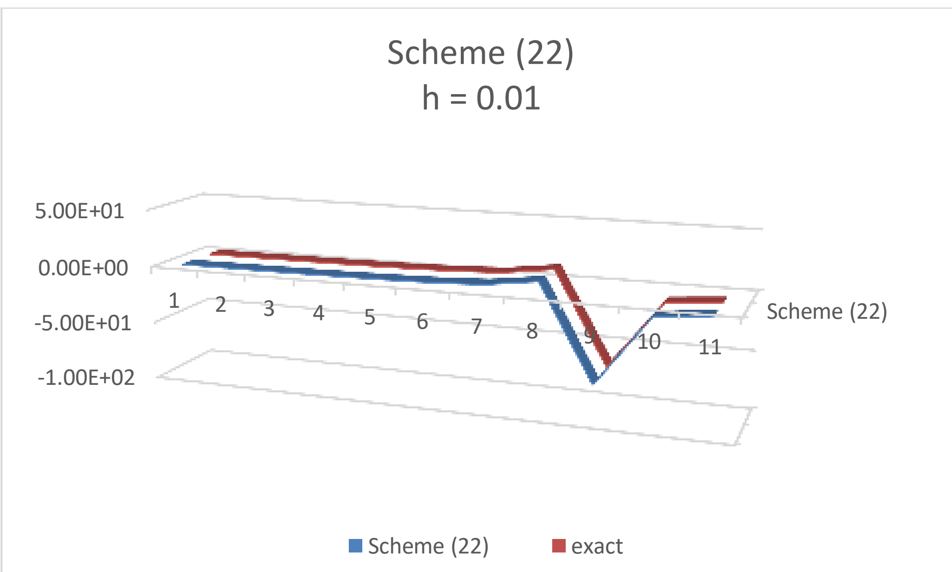


Figure 4. Graphical Representation of Scheme (22) on initial value problem 1 with step size h = 0.01

**Table 5**
Step size h = 0.1

| s/n | $x_n$ | Scheme (24) | exact | Error (24) |
|---|---|---|---|---|
| 1 | 0.10 | 0.12229185E+01 | 0.12238381E+01 | 0.91958046E-03 |
| 2 | 0.20 | 0.15077503E+01 | 0.15095335E+01 | 0.17832518E-02 |
| 3 | 0.30 | 0.18931659E+01 | 0.18972181E+01 | 0.40521622E-02 |
| 4 | 0.40 | 0.24569838E+01 | 0.24672010E+01 | 0.10217190E-01 |
| 5 | 0.50 | 0.33826470E+01 | 0.34122157E+01 | 0.29568672E-01 |
| 6 | 0.60 | 0.52288637E+01 | 0.53411722E+01 | 0.11230850E+00 |
| 7 | 0.70 | 0.10847114E+02 | 0.11724983E+02 | 0.87786961E+00 |
| 8 | 0.80 | 0.25621671E+03 | -0.67028618E+02 | 0.32324533E+03 |
| 9 | 0.90 | 0.20035935E+04 | -0.86635237E+01 | 0.20122571E+04 |
| 10 | 1.00 | 0.23498921E+04 | -0.45810776E+01 | 0.23544731E+04 |

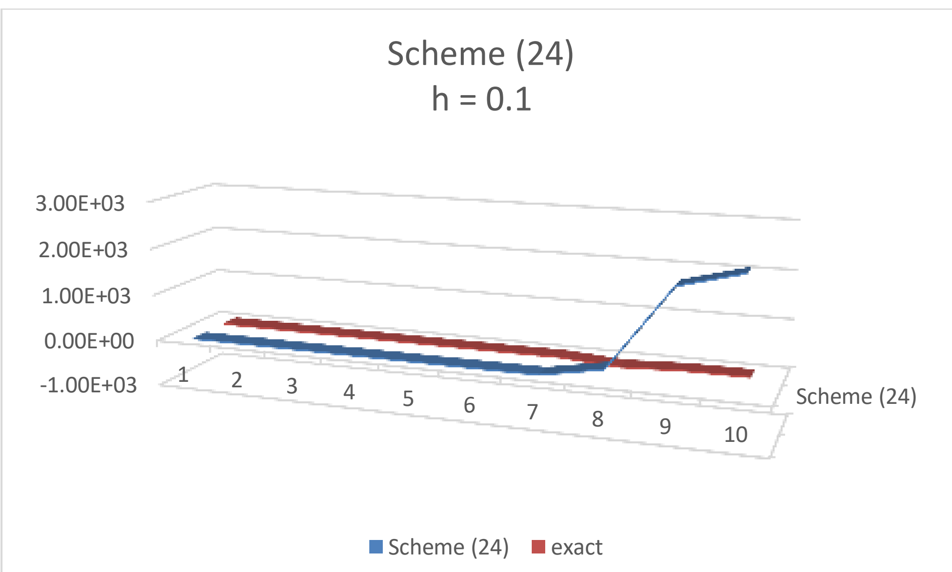


Figure 5. Graphical Representation of Scheme (24) on initial value problem 1 with step size h = 0.1.

**Table 6**
Step size h = 0.01

| s /n | $x_n$ | Scheme (24) | exact | Error (24) |
|---|---|---|---|---|
| 0 | 0.01 | 0.10202019E+01 | 0.10208480E+01 | 0.64611435E-03 |
| 1 | 0.10 | 0.12229185E+01 | 0.12238381E+01 | 0.91958046E-03 |
| 2 | 0.20 | 0.15077503E+01 | 0.15095335E+01 | 0.17832518E-02 |
| 3 | 0.30 | 0.18931659E+01 | 0.18972181E+01 | 0.40521622E-02 |
| 4 | 0.40 | 0.24569838E+01 | 0.24672010E+01 | 0.10217190E-01 |
| 5 | 0.50 | 0.33826470E+01 | 0.34122157E+01 | 0.29568672E-01 |
| 6 | 0.60 | 0.52288637E+01 | 0.53411722E+01 | 0.11230850E+00 |
| 7 | 0.70 | 0.10847114E+02 | 0.11724983E+02 | 0.87786961E+00 |
| 8 | 0.80 | 0.25621671E+03 | -0.67028618E+02 | 0.32324533E+0 |
| 9 | 0.90 | 0.20035935E+04 | -0.86635237E+01 | 0.20122571E+04 |
| 10 | 1.00 | 0.23498921E+04 | -0.45810776E+01 | 0.23544731E+04 |

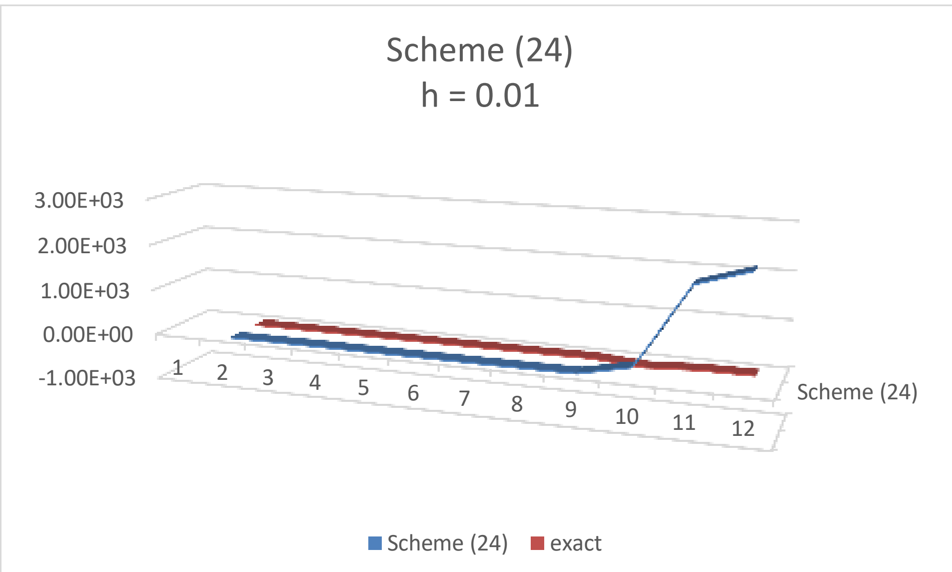


Figure 6. Graphical Representation of Scheme (24) on initial value problem 1 with step size h = 0.01.

Using nonstandard schemes (29), (30) and (31) to solve the initial value problem

2. $y' = -y^2,\ y(0) = 1$, in the interval $0 \le x \le 1$, the theoretical value is $y(x) = \dfrac{1}{1+x}$

**Table 7**

Step size h = 0.1

| s/n | $x_n$ | Scheme (29) | exact | Error (29) |
|---|---|---|---|---|
| 1 | 0.100 | 0.90392309E+00 | 0.90909094E+00 | 0.51678419E-02 |
| 2 | 0.200 | 0.82476324E+00 | 0.83333331E+00 | 0.85700750E-02 |
| 3 | 0.300 | 0.75840288E+00 | 0.76923078E+00 | 0.10827899E-01 |
| 4 | 0.400 | 0.70196277E+00 | 0.71428573E+00 | 0.12322962E-01 |
| 5 | 0.500 | 0.65336835E+00 | 0.66666669E+00 | 0.13298333E-01 |
| 6 | 0.600 | 0.61108673E+00 | 0.62500000E+00 | 0.13913274E-01 |
| 7 | 0.700 | 0.57396048E+00 | 0.58823532E+00 | 0.14274836E-01 |
| 8 | 0.800 | 0.54109919E+00 | 0.55555552E+00 | 0.14456332E-01 |
| 9 | 0.900 | 0.51180667E+00 | 0.52631581E+00 | 0.14509141E-01 |
| 10 | 1.000 | 0.48553050E+00 | 0.50000000E+00 | 0.14469504E-01 |

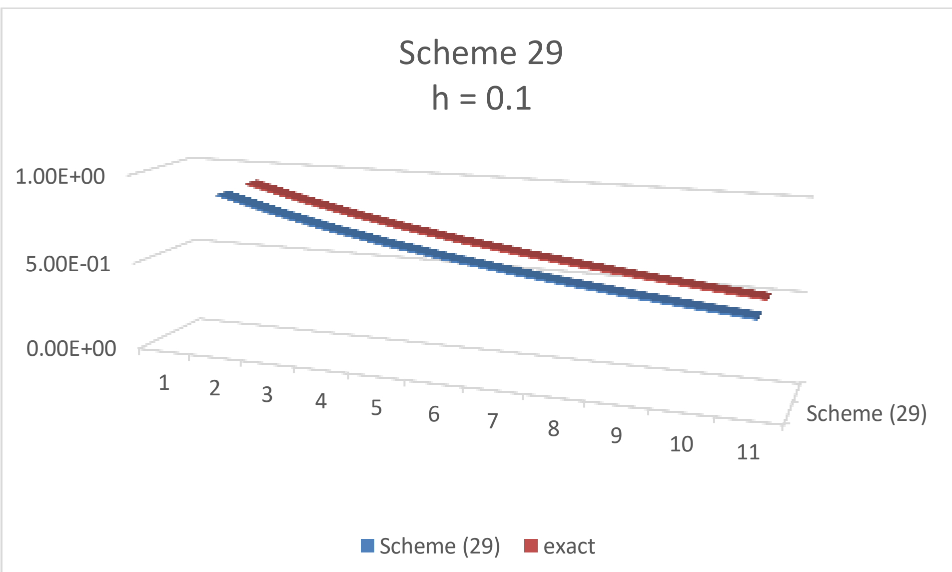


Figure 7. Graphical Representation of Scheme (29) on initial value problem 2 with step size h = 0.1.

**Table 8**
Step size h = 0.01

| s/n | $x_n$ | Scheme (29) | exact | Error (29) |
|---|---|---|---|---|
| 0 | 0.010 | 0.99003989E+00 | 0.99009901E+00 | 0.59127808E-04 |
| 1 | 0.100 | 0.90859658E+00 | 0.90909094E+00 | 0.49436092E-03 |
| 2 | 0.200 | 0.83250874E+00 | 0.83333331E+00 | 0.82457066E-03 |
| 3 | 0.300 | 0.76818395E+00 | 0.76923078E+00 | 0.10468364E-02 |
| 4 | 0.400 | 0.71308959E+00 | 0.71428573E+00 | 0.11961460E-02 |
| 5 | 0.500 | 0.66537148E+00 | 0.66666669E+00 | 0.12952089E-02 |
| 6 | 0.600 | 0.62364089E+00 | 0.62500000E+00 | 0.13591051E-02 |
| 7 | 0.700 | 0.58683723E+00 | 0.58823532E+00 | 0.13980865E-02 |
| 8 | 0.800 | 0.55413657E+00 | 0.55555558E+00 | 0.14190078E-02 |
| 9 | 0.900 | 0.52488869E+00 | 0.52631581E+00 | 0.14271140E-02 |
| 10 | 1.000 | 0.49857432E+00 | 0.50000000E+00 | 0.14256835E-02 |

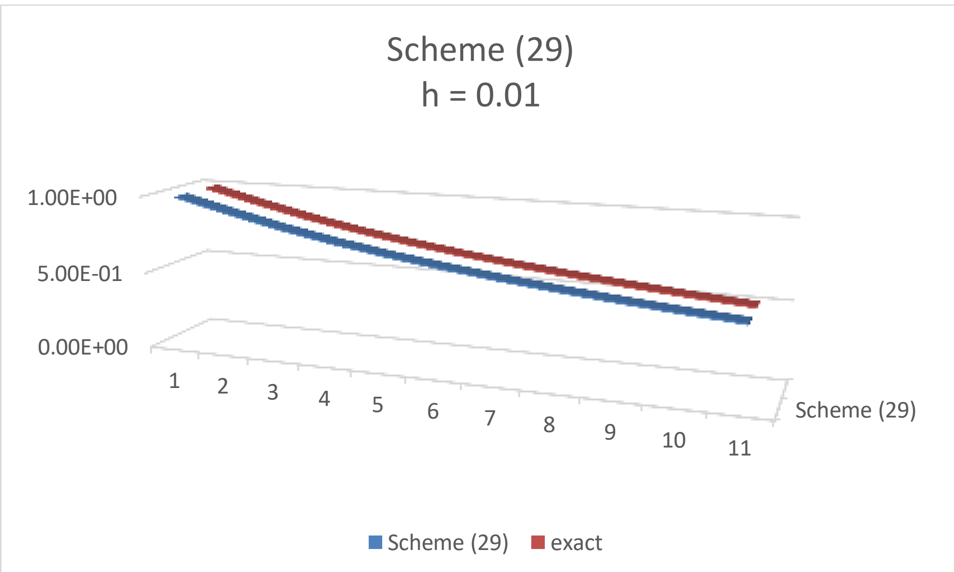


Figure 8. Graphical Representation of Scheme (29) on initial value problem 2 with step size h = 0.01.

**Table 9**
Step size h = 0.1

| s/n | $x_n$ | Scheme (30) | exact | Error (30) |
|---|---|---|---|---|
| 1 | 0.100 | 0.90483737E+00 | 0.90909094E+00 | 0.42535663E-02 |
| 2 | 0.200 | 0.82621282E+00 | 0.83333331E+00 | 0.71204901E-02 |
| 3 | 0.300 | 0.76015979E+00 | 0.76923078E+00 | 0.90709925E-02 |
| 4 | 0.400 | 0.70388639E+00 | 0.71428573E+00 | 0.10399342E-01 |
| 5 | 0.500 | 0.65537035E+00 | 0.66666669E+00 | 0.11296332E-01 |
| 6 | 0.600 | 0.61311108E+00 | 0.62500000E+00 | 0.11888921E-01 |
| 7 | 0.700 | 0.57597160E+00 | 0.58823532E+00 | 0.12263715E-01 |
| 8 | 0.800 | 0.54307461E+00 | 0.55555552E+00 | 0.12480915E-01 |
| 9 | 0.900 | 0.51373243E+00 | 0.52631581E+00 | 0.12583375E-01 |
| 10 | 1.000 | 0.48739842E+00 | 0.50000000E+00 | 0.12601584E-01 |

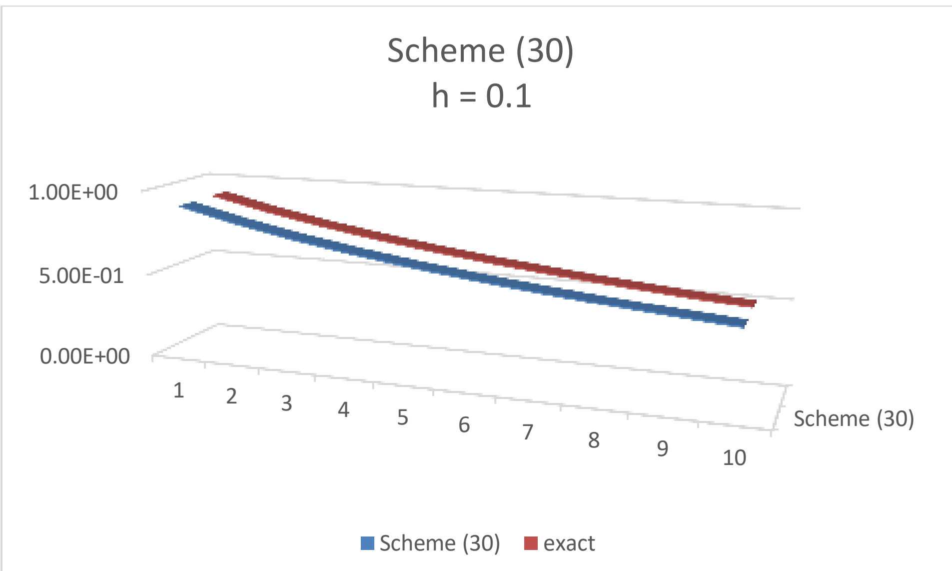


Figure 9. Graphical Representation of Scheme (30) on initial value problem 2 with step size h = 0.1.

**Table 10**
Step size h = 0.01

| s/n | $x_n$ | Scheme(30) | exact | Error (30) |
|---|---|---|---|---|
| 0 | 0.010 | 0.99004984E+00 | 0.99009901E+00 | 0.49173832E-04 |
| 1 | 0.100 | 0.90867639E+00 | 0.90909094E+00 | 0.41455030E-03 |
| 2 | 0.200 | 0.83263701E+00 | 0.83333331E+00 | 0.69630146E-03 |
| 3 | 0.300 | 0.76834106E+00 | 0.76923078E+00 | 0.88971853E-03 |
| 4 | 0.400 | 0.71326309E+00 | 0.71428573E+00 | 0.10226369E-02 |
| 5 | 0.500 | 0.66555345E+00 | 0.66666669E+00 | 0.11132360E-02 |
| 6 | 0.600 | 0.62382609E+00 | 0.62500000E+00 | 0.11739135E-02 |
| 7 | 0.700 | 0.58702230E+00 | 0.58823532E+00 | 0.12130141E-02 |
| 8 | 0.800 | 0.55431926E+00 | 0.55555558E+00 | 0.12363195E-02 |
| 9 | 0.900 | 0.52506769E+00 | 0.52631581E+00 | 0.12481213E-02 |
| 10 | 1.000 | 0.49874866E+00 | 0.50000000E+00 | 0.12513399E-02 |

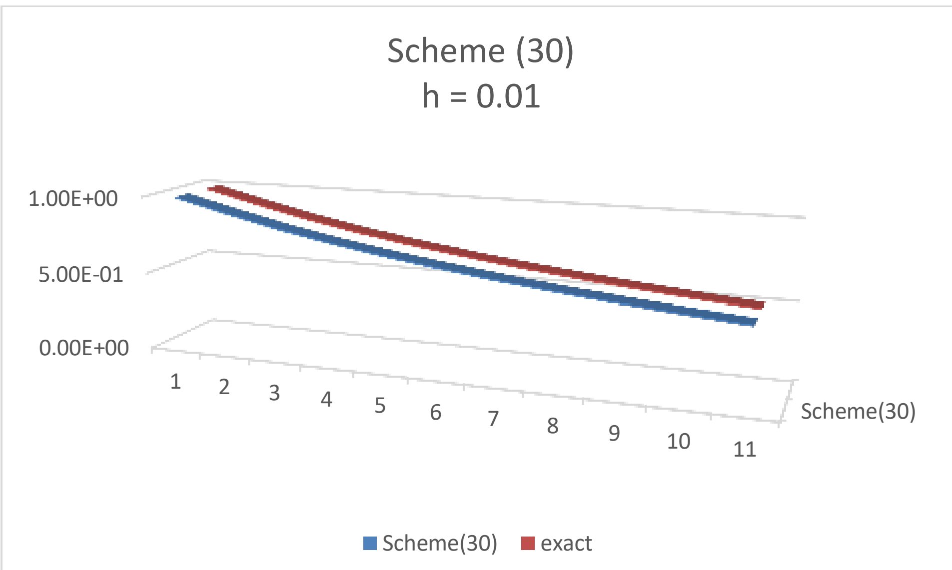


Figure 10. Graphical Representation of Scheme (30) on initial value problem 2 with step size h = 0.01.

**Table 11**
Step size h = 0.1

| s/n | $x_n$ | Scheme (31) | exact | Error (31) |
|---|---|---|---|---|
| 1 | 0.100 | 0.90008318E+00 | 0.90909094E+00 | 0.90077519E-02 |
| 2 | 0.200 | 0.81872952E+00 | 0.83333331E+00 | 0.14603794E-01 |
| 3 | 0.300 | 0.75114143E+00 | 0.76923078E+00 | 0.18089354E-01 |
| 4 | 0.400 | 0.69405735E+00 | 0.71428573E+00 | 0.20228386E-01 |
| 5 | 0.500 | 0.64517879E+00 | 0.66666669E+00 | 0.21487892E-01 |
| 6 | 0.600 | 0.60283732E+00 | 0.62500000E+00 | 0.22162676E-01 |
| 7 | 0.700 | 0.56579125E+00 | 0.58823532E+00 | 0.22444069E-01 |
| 8 | 0.800 | 0.53309667E+00 | 0.55555552E+00 | 0.22458851E-01 |
| 9 | 0.900 | 0.50402296E+00 | 0.52631581E+00 | 0.22292852E-01 |
| 10 | 1.000 | 0.47799525E+00 | 0.50000000E+00 | 0.22004753E-01 |

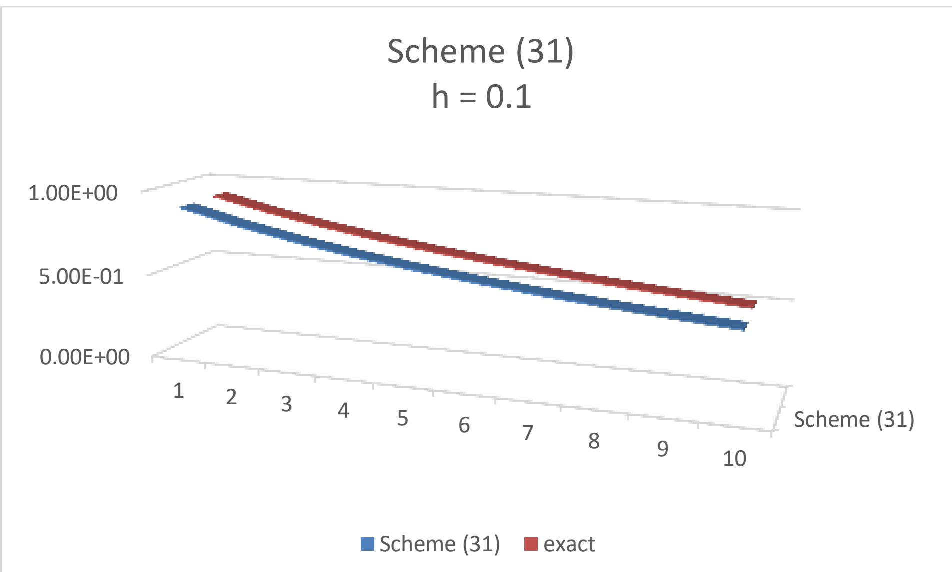


Figure 11 Graphical Representation of Scheme (31) on initial value problem 2 with step size h = 0.1.

**Table 12**
Step size h = 0.01

| s/n | $x_n$ | Scheme (31) | exact | Error (31) |
|---|---|---|---|---|
| 0 | 0.010 | 0.99000007E+00 | 0.99009901E+00 | 0.98943710E-04 |
| 1 | 0.100 | 0.90827549E+00 | 0.90909094E+00 | 0.81545115E-03 |
| 2 | 0.200 | 0.83199376E+00 | 0.83333331E+00 | 0.13395548E-02 |
| 3 | 0.300 | 0.76755363E+00 | 0.76923078E+00 | 0.16771555E-02 |
| 4 | 0.400 | 0.71239358E+00 | 0.71428573E+00 | 0.18921494E-02 |
| 5 | 0.500 | 0.66464168E+00 | 0.66666669E+00 | 0.20250082E-02 |
| 6 | 0.600 | 0.62289822E+00 | 0.62500000E+00 | 0.21017790E-02 |
| 7 | 0.700 | 0.58609527E+00 | 0.58823532E+00 | 0.21400452E-02 |
| 8 | 0.800 | 0.55340397E+00 | 0.55555558E+00 | 0.21516085E-02 |
| 9 | 0.900 | 0.52417129E+00 | 0.52631581E+00 | 0.21445155E-02 |
| 10 | 1.000 | 0.49787548E+00 | 0.50000000E+00 | 0.21245182E-02 |

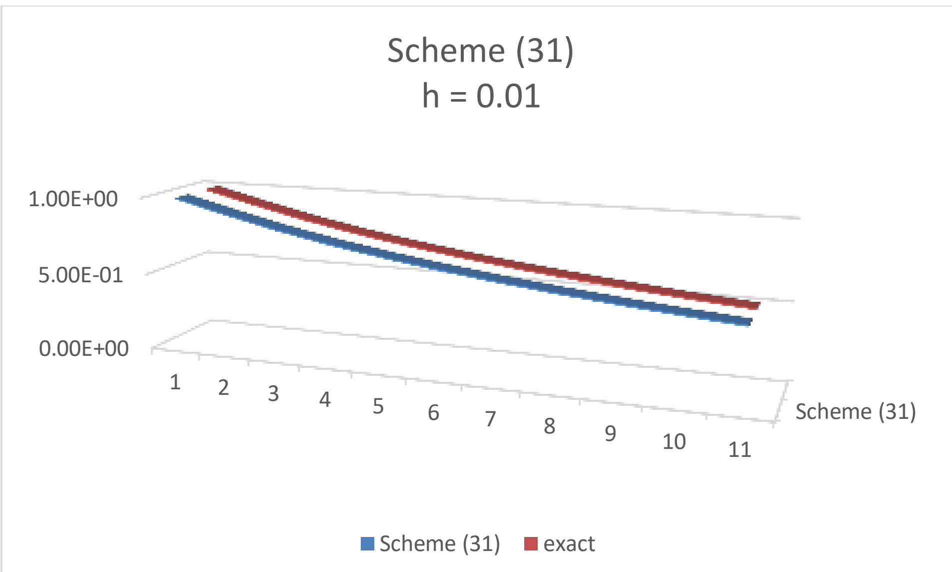


Figure 12. Graphical Representation of Scheme (31) on initial value problem 2 with step size h = 0.01.

## 4. Conclusion

In this paper we have been able to develop nonstandard finite difference schemes for the solution of autonomous first order differential equations. It was discovered from the implementation of the numerical schemes that scheme (22) performs better than the two other schemes in that it mimicked the theoretical solution of the differential equation $y' = 1 + y^2$ very well, but the three schemes (29), (30) and (31) works well on $y' = -y^2$. Hence we have been able to establish the strength, efficiency and the qualitative stability properties of the nonstandard schemes.

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