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On the relative mix transition probabilities in repairman problem of two different types with batch deterministic repairs

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Abstract

In this paper, the relative mix in the distribution of each machine type and the link between the durability and maintenance of each type of machine, are presented. Compared to existing machine repair problem, this study considered the repairman problem with multiple batch deterministic repairs of two different machine types A_1 and A_2. Without loss of generality, it is assumed that, there are integer $k_1 = 2$ of type A_1 machines and integer $k_2 = 2$ of type A_2 machines in the system such that $k_1 + k_2 = k$. Each type A_1 (A_2) machine occasionally breaks down and moves into repair queue Q_1 (Q_2) at Poisson rate $\lambda_1(\lambda_2)$. The Repair station has a capacity to repair at most two machines in a session. Flow balance equations are obtained for each of the 22 states of the system as defined by observing the system at repair time points. The resulting equations were solved for stationary probabilities at repair point and some server occupancy mode conditions using Gaussian elimination. Steady state repair completion mix probabilities $(p_1, p_2, p_3, \cdots, p_{22})$ and steady state occupancy probabilities $q_j: j = 1,2,\cdots,7$ are obtained. Subsequently, the work are evaluated with adopted arrival rates $\lambda_1 = 0.25, 0.50, 0.75, 1.00, \cdots, 2.50$, $\lambda_2 = 2.00$, and specific deterministic repair times D_{st} for s and t units of type A_1 and A_2 machines respectively. Using the Minkwoski distance of the various runs and system configuration as a guide, it was established that the closer the system is to a balanced situation the less is the discrepancy in the relative mix of the state probability distribution. We concluded that the relative mix can be rather different from each other depending on the appropriate parameter for the model.

Keywords: Machine repair problem; Flow balance equations; Occupancy mode condition; Relative mix.

1.0 Introduction

The performance of any machining system plays a vital role in human life. This has pervaded every field of our lives in different activities ensuring our almost total dependence on machines. However, as the time passes, machines become prone to failure. The failure of machines may result to loss of production, money, goodwill and inconvenience, among others, if at any time a machine fails, it is sent to the repair facility for necessary repair. To avoid the aforementioned loss in machining system, the spare and appropriate repair facility should be incorporated. In view of such a design, the standby units play an important role - so that the machining system may keep working to provide the desired grade of service at all times. If a machine fails, an available standby unit replaces it and the failed machine is sent for immediate repair. A standby unit may be cold standby type which has zero failure rate. It may also be warm standby that has non-zero failure rate . Today's machining systems are highly sophisticated and complex. They comprise a number of complicated parts. Failure of any part(s) or whole machine directly affects the service system being sought. Thus, a machining system can be out of order at any stage or can have different reasons or models for its failure.

Notation

A_i: Machine type i, i=1,2.

n_i: The number of failed units of A_i $i = 1,2$ machines on the repair queue.

k_i: Total number of A_i $i = 1,2$; $k_1 + k_2 = k$.

Q_i: Repair queue of A_i.

D_{st}: Constant deterministic time taking if there is s type of A_1 and t of type A_2 : $s, t \epsilon Z^+$.

$n_i{}^*$: The number of failed units of A_i $i = 1,2$ machines at the feasible state

λ_i**:** Failure rate of type A_i: $i = 1,2$ machines.

μ_i: Repair rate of type A_i: $i = 1,2$ machines.

P_n: Probability that there are n failed units present in the system in steady state

k: Total number of machines in the system

2.0 Mathematical model

Consider a closed system of k machines that are either working or under repairs.

Suppose there are integer k_1 type A_1 machines and integer k_2 type A_2 machines in the system such that $k_1 + k_2 = k$. Suppose each type A_1 machine occasionally breaks down and moves into repair queue Q_1 at Poisson rate λ_1. Suppose also that each type A_2 machine occasionally breaks down and moves into repair queue Q_2 at Poisson rate λ_2. The Repair station has a capacity to repair at most two machines in a session. Subject to the availability of machine on the repair queues, the repair manager may pick the two for repairs from Q_1 or the two from Q_2. It may also choose to pick one from either Q_1 and Q_2 . The random number of machine chosen from Q_1 is given by the Binomial distribution $B(2, \tau)$, where τ is the probability of randomly choosing a machine from Q_1. This is equivalent, from the point of view of choosing from Q_2, to the distribution $B(2,1 - \tau)$, where $1 - \tau$ is the probability of randomly choosing a machine from Q_2. We assume that repair takes a constant deterministic time D_{11} if there is one of each machine type in repair. We also assume that repair takes a constant deterministic time D_{20} if the two machines are of type A_1. and repair take a constant deterministic time D_{02} if the two machines are of type A_2. In the event that only one queue is occupied, two of the machine types are served at D_{20} or D_{02} and each available machine for repair in Q_1 or Q_2 is served at constant timeD_{10} or D_{01} depending on which type is available or not available.

Our main interest is in the distribution of the mix in number of each of machine type on the repair queues. Noting that the system is not a straight forward Markovian system, we examine the distribution mix at repair completion times.

2.1 State definition

Suppose (n_1, n_2, i) denotes the state that there are integer n_1 type A_1 machines and integer n_2 type A_2 machines on the repair queue at the moment just after a repair completion of type $i, i = 0,1,2,3,4$ as discussed in (Baba, 2012)

Suppose $p(n_1, n_2, i)$ denote the probability that there are integer n_1 type A_1 machines and integer n_2 type A_2 machines on the repair queue at the moment just after a repair completion of type $i, i = 0,1,2,3,4$.

Let $n = n_1 + n_2$.

The symbol i takes the value of 0 if the repaired consists of one of the type A_1 machine and one of the type A_2 machines.

It takes the value 1 if two of the type A_1 are repaired and the value 2 if two of the type A_2 are repaired.

It takes the value 3 if only one of type A_1 is repaired and value 4 if only one of type A_2 is repaired.

The schematic diagram to represent the Repairman Problem with Multiple Batch

2.2 Enumeration of states (Obilade, 1990)

We note as follows:

$0 < n_1 < k_1 - 1$ and $0 < n_2 < k_2 - 1$ if $i = 0$

$0 < n_1 < k_1 - 2$ and $0 < n_2 < k_2$ if $i = 1$

$0 < n_1 < k_1$ and $0 < n_2 < k_2 - 2$ if $i = 2$

$0 < n_1 < k_1 - 1$ and $0 < n_2 < k_2$ if $i = 3$

$0 < n_1 < k_1$ and $0 < n_2 < k_2 - 1$ if $i = 4$

The total number of possible states is calculated as follows:

Deterministic Repairs is shown in Fig. 1:

$$k_1k_2 + (k_1 - 1)(k_2 + 1) + (k_1 + 1)(k_2 - 1) + k_1(k_2 + 1) + (k_1 + 1)k_2$$

$$= 5k_1k_2 + k_1 + k_2 - 2$$

Thus, if $k_1 = k_2 = 2$, the total number of possible states is 22.

The chronological listing of the 22 possible states is given in Table 1 for the specific case of $k_1 = k_2 = 2$. The table also contains the pseudonyms (Sharma, 2012):

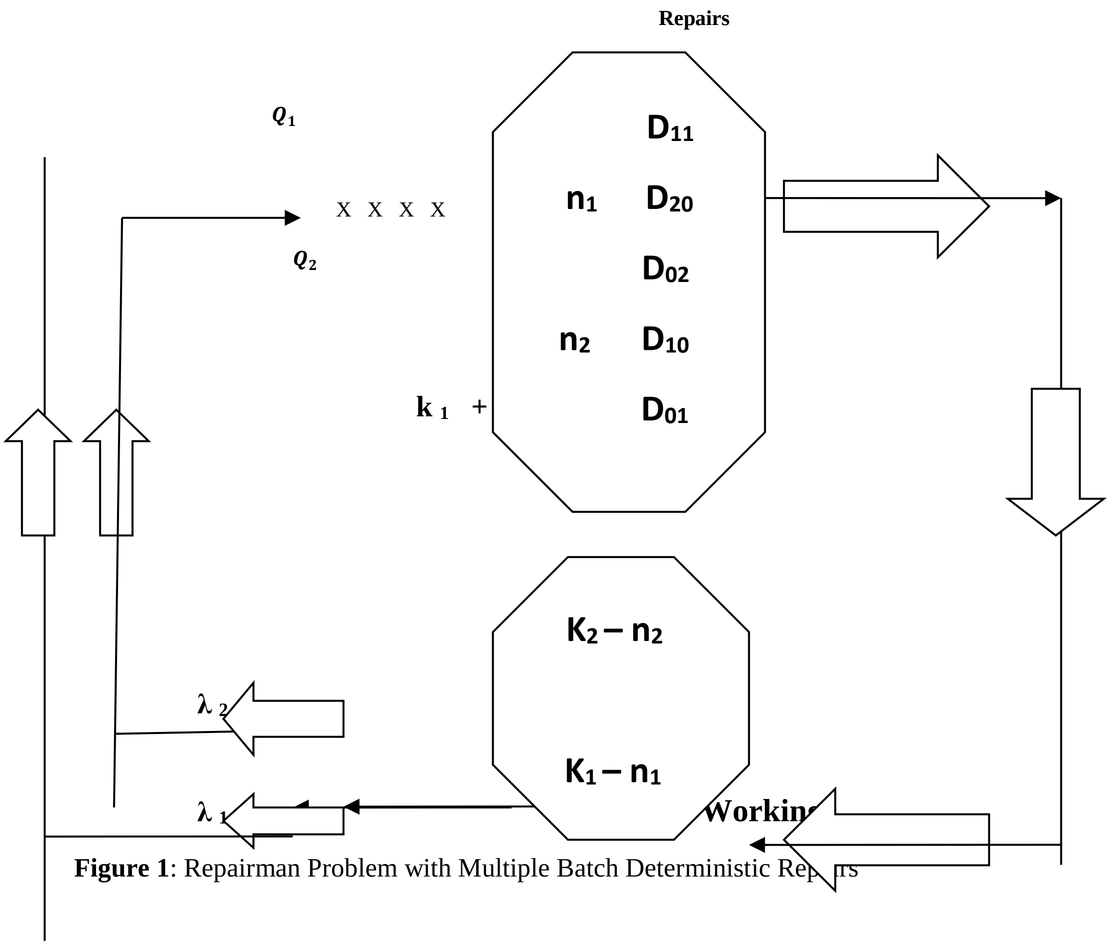

Figure 1: Repairman Problem with Multiple Batch Deterministic Repairs

Table 1: The Chronological Order of the States for case $k_1 = k_2 = 2$

States	Pseudonyms	States	Pseudonyms	States	Pseudonyms
0 0 0	(1)	1 0 0	(12)	2 0 2	(20)
0 0 1	(2)	1 0 2	(13)	2 0 4	(21)
0 0 2	(3)	1 0 3	(14)	2 1 4	(22)
0 0 3	(4)	1 0 4	(15)		
0 0 4	(5)	1 1 0	(16)		
0 1 0	(6)	1 1 3	(17)		
0 1 1	(7)	1 1 4	(18)		
0 1 3	(8)	1 2 3	(19)		
0 1 4	(9)				
0 2 1	(10)				
0 2 3	(11)				

By way of example the state 0 2 3 (also called state 11) represents the state when there is no type A_1 machine in repair but two type A_2 machine in repair immediately after the completion of only one type A_1 machine
.

Table 2: Feasible transition points from $(n_1;\ n_2;\ i)$ to (n_1^*, n_2^*, j) for case $k_1 = k_2 = 2$
(Jain *et al*, 2016)

(n_1^*, n_2^*, j) / (n_1, n_2, i)	0 0 0	0 0 1	00 2	00 3	0 0 4	0 1 0	01 1	01 3	0 1 4	0 2 1	0 2 3	1 0 0	1 0 2	1 0 3	10 4	11 0	1 1 3	11 4	1 2 3	20 2	20 4	21 4
0 0 0				*	*			*	*		*			*	*		*	*	*		*	*
0 0 1				*	*			*	*		*			*	*		*	*	*		*	*
0 0 2				*	*			*	*		*			*	*		*	*	*		*	*
0 0 3				*	*			*	*		*			*	*		*	*	*		*	*
0 0 4				*	*			*	*		*			*	*		*	*	*		*	*
0 1 0					*				*						*			*			*	*
0 1 1					*				*						*			*			*	*
0 1 3					*				*						*			*			*	*
0 1 4					*				*						*			*			*	*
0 2 1			*										*							*		
0 2 3			*										*							*		
1 0 0				*				*			*			*			*		*			
1 0 2				*				*			*			*			*		*			
1 0 3				*				*			*			*			*		*			
1 0 4				*				*			*			*			*		*			
1 1 0	*					*						*				*						
1 1 3	*					*						*				*						
1 1 4	*					*						*				*						
1 2 3						*							*			*				*		
2 0 2		*					*			*												
2 0 4		*					*			*												
2 1 4							*			*		*				*						

Table 2 gives the feasible transition points. By way of illustration, transition into state 0 2 3 is possible from 0 0 0, 0 0 1, 0 0 2, 0 0 3, 0 0 4 as well as 1 0 0, 1 0 2, 1 0 3 and 1 0 4. The same way, a transition from state 0 2 3 may led into 0 0 2, 1 0 2 and 2 0 2 only.

We present two samples of transition probabilities from each state $(n_1;\ n_2;\ i)$ into the feasible state (n_1^*, n_2^*, j) out of 22 as follows:

2.3 Sample Transit Problem

From state 000, we can move into the following destination states with the attached probabilities (Dewilde, 2013):

---- 0 0 3 $:= \frac{\lambda_1}{\lambda}[\exp(-\lambda_1 D_{10})][\exp(-\lambda_2 D_{10})]$

---- 0 0 4 $:= \frac{\lambda_2}{\lambda}[\exp(-\lambda_1 D_{01})][\exp(-\lambda_2 D_{01})]$

---- 0 1 3 $:= \frac{\lambda_1}{\lambda}[\lambda_2 D_{10}\exp(-\lambda_2 D_{10})][\exp(-\lambda_1 D_{10})]$

---- 0 1 4 $:= \frac{\lambda_2}{\lambda}[1-\exp(-\lambda_2 D_{01})][\exp(-\lambda_1 D_{01})]$

---- 0 2 3 $:= \frac{\lambda_1}{\lambda}[\lambda_1 D_{10}\exp(-\lambda_1 D_{10})][1-\exp(-\lambda_2 D_{10})-\lambda_2 D_{10}\exp(-\lambda_2 D_{10})]$

---- 1 0 3 $:= \frac{\lambda_1}{\lambda}[1-\exp(-\lambda_1 D_{10})][\exp(-\lambda_2 D_{10})]$

---- 1 0 4 $:= \frac{\lambda_2}{\lambda}[\lambda_1 D_{01}\exp(-\lambda_1 D_{01})][\exp(-\lambda_2 D_{01})]$

---- 1 1 3 $:= \frac{\lambda_1}{\lambda}[1-\exp(-\lambda_1 D_{10})][\lambda_2 D_{10}\exp(-\lambda_2 D_{10})]$

---- 1 1 4 $:= \frac{\lambda_2}{\lambda}[\lambda_1 D_{01}\exp(-\lambda_1 D_{01})][1-\exp(-\lambda_2 D_{01})]$

---- 1 2 3 $:= \frac{\lambda_1}{\lambda}[1-\exp(-\lambda_1 D_{10})][1-\exp(-\lambda_2 D_{10})(1+\lambda_2 D_{10})]$

---- 2 0 4 $:= \frac{\lambda_2}{\lambda}[1-\exp(-\lambda_1 D_{10})(1+\lambda_1 D_{01})][\exp(-\lambda_2 D_{01})]$

---- 2 1 4 $:= \frac{\lambda_2}{\lambda}[1-\exp(-\lambda_1 D_{10})(1+\lambda_1 D_{01})][1-\exp(-\lambda_2 D_{01})]$

The same for states 0 0 1, 0 0 2, 0 0 3 and 0 0 4.

From state 0 1 0, we can move to the following destination states with the following attached probabilities.

----0 0 4 $:= [\exp(-\lambda_1 D_{01})][\exp(-\lambda_2 D_{01})]$

----0 1 4 $:= [\exp(-\lambda_1 D_{01})][1-\exp(-\lambda_2 D_{01})]$

---- 1 0 4 $:= [\lambda_1 D_{01}\exp(-\lambda_1 D_{01})][\exp(-\lambda_2 D_{01})]$

---- 1 1 4 $:= [\lambda_1 D_{01}\exp(-\lambda_1 D_{01})][1-\exp(-\lambda_2 D_{01})]$

---- 2 0 4 $:= [1-\exp(-\lambda_1 D_{10})(1+\lambda_1 D_{01})][\exp(-\lambda_2 D_{01})]$

---- 2 1 4 $:= [1-\exp(-\lambda_1 D_{10})(1+\lambda_1 D_{01})][1-\exp(-\lambda_2 D_{01})]$

The same for states 0 1 1, 0 1 3 and 0 1 4.

2.4 System of linear equations for the transition probabilities for $k_1 = k_2 = 2$

Notations

We suppose the following notations:

$$a_{ij} = \exp(-\lambda_1 D_{ij}) \quad (1)$$

$$b_{ij} = \exp(-\lambda_2 D_{ij})$$

With $ij \in \{(1,1),(2,0),(1,0),(0,1),(0,2)\}$ (Jain *et al*, 2014), the flow balance equations for the system are generally given as

$$p_1 = a_{11}b_{11}(p_{16}+p_{17}+p_{18}) \quad (2)$$

$$p_2 = b_{20}(p_{20}+p_{21}) \quad (3)$$

$$p_3 = a_{02}(p_{10}+p_{11}) \quad (4)$$

$$p_4 = a_{10}b_{10}[\tfrac{\lambda_1}{\lambda}(p_1+p_2+p_3+p_4+p_5)+(p_{12}+p_{13}+p_{14}+p_{15})] \quad (5)$$

$$p_5 = \tfrac{\lambda_2}{\lambda}a_{01}b_{01}(p_1+p_2+p_3+p_4+p_5)+a_{22}b_{22}(p_6+p_7+p_8+p_9) \quad (6)$$

$$p_6 = a_{11}(1-b_{11})(p_{16}+p_{17}+p_{18})+\tfrac{1}{2}a_{11}p_{19} \quad (7)$$

$$p_7 = \lambda_2 D_{20}b_{20}(p_{20}+p_{21})+\tfrac{1}{2}b_{20}p_{22} \quad (8)$$

$$p_8 = a_{10}\lambda_2 D_{10}b_{10}[\tfrac{\lambda_1}{\lambda}(p_1+p_2+p_3+p_4+p_5)+(p_{12}+p_{13}+p_{14}+p_{15})] \quad (9)$$

$$p_9 = a_{01}(1-b_{01})[(p_1+p_2+p_3+p_4+p_5)+(p_6+p_7+p_8+p_9)] \quad (10)$$

$$p_{10} = (1-b_{20}-\lambda_2 D_{20}b_{20})(p_{20}+p_{21})+\tfrac{1}{2}(1-b_{20})p_{22} \quad (11)$$

$$p_{11} = a_{01}(1-\lambda_2 b_{10}D_{10}b_{10})[\tfrac{\lambda_1}{\lambda}(p_1+p_2+p_3+p_4+p_5)+(p_{12}+p_{13}+p_{14}+p_{15}] \quad (12)$$

$$p_{12} = (1-a_{11})b_{11}(p_{16}+p_{17}+p_{18})+\tfrac{1}{2}b_{20}p_{22} \quad (13)$$

$$p_{13} = \lambda_1 D_{02}a_{02}(p_{10}+p_{11})+\tfrac{1}{2}a_{02}p_{19} \quad (14)$$

$$p_{14} = (1-a_{10})b_{10}[\tfrac{\lambda_1}{\lambda}(p_1+p_2+p_3+p_4+p_5)+(p_{12}+p_{13}+p_{14}+p_{15})] \quad (15)$$

$$p_{15} = D_{01}a_{01}b_{01}[\tfrac{\lambda_2}{\lambda}(p_1+p_2+p_3+p_4+p_5)+(p_6+p_7+p_8+p_9)] \quad (16)$$

$$p_{16} = (1-a_{11})(1-b_{11})(p_{16}+p_{17}+p_{18})+\tfrac{1}{2}(1-a_{11})p_{19}+\tfrac{1}{2}(1-b_{20})p_{22} \quad (17)$$

$$p_{17} = \lambda_2(1-a_{10})D_{10}b_{10}[\tfrac{\lambda_1}{\lambda}(p_1+p_2+p_3+p_4+p_5)+(p_{12}+p_{13}+p_{14}+p_{15})] \quad (18)$$

$p_{18} = \lambda_1(1 - b_{01}) D_{01}a_{01}[\frac{\lambda_2}{\lambda}(p_1 + p_2 + p_3 + p_4 + p_5) + (p_6 + p_7 + p_8 + p_9)]$ (19)

$p_{19} = (1 - a_{10})(1 - b_{10} - \lambda_2 D_{10} b_{10})[\frac{\lambda_1}{\lambda}(p_1 + p_2 + p_3 + p_4 + p_5) + (p_{12} + p_{13} + p_{14} + p_{15})]$ (20)

$p_{20} = (1 - a_{02} - \lambda_1 D_{02} a_{02})(p_{10} + p_{11}) + \frac{1}{2}(1 - a_{02})p_{19}$ (21)

$p_{21} = b_{01}(1 - a_{01} - \lambda_1 D_{01} a_{01})[\frac{\lambda_2}{\lambda}(p_1 + p_2 + p_3 + p_4 + p_5) + (p_6 + p_7 + p_8 + p_9)]$ (22)

$p_{22} = (1 - b_{01})(1 - a_{01} - \lambda_1 D_{01} a_{01})[\frac{\lambda_2}{\lambda}(p_1 + p_2 + p_3 + p_4 + p_5) + (p_6 + p_7 + p_8 + p_9)]$ (23)

where, as expected of probabilities,

$$p_1 + p_2 + p_3 + p_4 + \cdots + p_{22} = 1$$

Let q_1 denotes probability of repair queue being empty immediately following a completion of repair. i.e. no unit under repairs (all are working),

Therefore

$$q_1 = p_1 + p_2 + p_3 + p_4 + p_5 \quad \ldots\ldots\ldots (24)$$
$$q_2 = p_6 + p_7 + p_8 + p_9 \quad \ldots\ldots\ldots (25)$$
$$q_3 = p_{10} + p_{11} \quad \ldots\ldots\ldots (26)$$
$$q_4 = p_{12} + p_{13} + p_{14} + p_{15} \quad (27)$$
$$q_5 = p_{16} + p_{17} + p_{18} \quad \ldots (28)$$
$$q_6 = p_{20} + p_{21} \quad \ldots\ldots\ldots (29)$$
$$q_7 = p_{19} + p_{22} \quad \ldots\ldots\ldots (30)$$

of course, q_7 denotes the probability of repair queue being occupied by three units of machines, one of type A_1 and two of type A_2 or two of type A_1 and one of type A_2. The flow balance equations for the system are generally given by:

Summing Equations $(2), \cdots, (6)$, we have:

$[\frac{\lambda_1}{\lambda} D_{10}b_{10}b_{10} + \frac{\lambda_2}{\lambda} a_{01}b_{01} - 1]q_1 + a_{01}b_{01}q_2 + a2q_3 + a_{10}b_{10}q_4 + a_{11}b_{11}q_5 + b_{20}q_6 = 0$ (31)

Similarly, summing Equations $(7), \cdots, (10)$, we have:

$[\frac{\lambda_1}{\lambda} a_{10}\lambda_2 D_{10}b_{10} + \frac{\lambda_2}{\lambda} a_{01}(1 - b_{01})]q_1 + [a_{01}(1 - b_{01}) - 1]q_2 + a_{10}\lambda_2 D_{10}b_{10}q_4 + a_{11}(1 - b_{11})q_5 + \lambda_2 D_{20}b_{20}q_6 = [a_{11}p_{19} + b_{02}p_{22}]$ (32)

Similarly, summing Equations (11) *and* (12), we have:

$\frac{\lambda_1}{\lambda} a_{10}(1 - b_{10} - \lambda_2 D_{10}b_{10})q_1 - q_3 + a_{10}(1 - b_{10} - \lambda_2 D_{10}b_{10})q_4 + (1 - b_{20} - \lambda_2 D_{20}b_{20})q_6 = -\frac{1}{2}[1 - b_{20}]p_{22}$ (33)

Similarly, summing Equations $(13), \cdots, (16)$, we have:

$[\frac{\lambda_1}{\lambda}(1 - a_{10})b_{10} + \frac{\lambda_2}{\lambda} D_{01}a_{01}b_{01}\lambda_1]q_1 + \lambda_1 D_{01}a_{01}b_{01}q_2 + \lambda_1 D_{02}a_{02}q_3 + [(1 - a_{10})b_{10} - 1]q_4 + (1 - a_{11})b_{11}q_5 = 0$ (34)

Similarly, summing Equations $(17), \cdots, (19)$, we have:

$\left[\frac{\lambda_1}{\lambda}(1 - a_{10})\lambda_2 D_{10}b_{10} + \frac{\lambda_2}{\lambda}(1 - b_{01})\lambda_1 D_{01}a_{01}\right]q_1 + (1 - b_{01})\lambda_2 D_{01}a_{01}q_2 + (1 - a_{10})\lambda_2 D_{10}b_{10}q_4 + (1 - a_{11})(1 - b_{11})q_5 = -\frac{1}{2}[(1 - a_{11})p_{19} + (1 - b_{20})p_{22}]$ (35)

Similarly, summing Equations $(20), \cdots, (23)$, we have:

$[\frac{\lambda_1}{\lambda}(1 - a_{10})(1 - \lambda_2 b_{10}D_{10}b_{10}) + \frac{\lambda_2}{\lambda} b_{01}(1 - a_{01} - \lambda_1 D_{01}a_{01})]q_1 + [1 - a_{01} - \lambda_1 D_{01}a_{01}]q_2 + (1 - a_{02} - \lambda_1 D_{02}a_{02})q_3 + (1 - a_{10})(1 - \lambda_2 b_{10}D_{10}b_{10})q_4 - q_6 = [1 - \frac{1}{2} + \frac{a02}{2}]p_{19} + p_{22}$ (36)

The normalizing equation is given as :

$q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + p_{19} + p_{22} = 1$ (37)

3.0 Numerical illusration

Equations (31) to (37) are amenable to solution by the Gaussian elimination method and other methods for solving a system of equation. Adopting some realistic values (Table 3) for the parameter as relevant to the application discussed in (Kleinrock, 1980), it is fairly straight forward to obtain the results given on Table 4 and 5 for the steady states probabilities p_j and occupancy probabilities q_j respectively. A close look at Table 3 will identify Run 8 has one with a balanced breakdown rates for the two machine types.

Table 3: Parameters Values Adopted for Various Runs

Runs	λ_1	λ_2	D_0	D_1	D_2	D_{11}	D_{22}
1	0.25	2	$\frac{11}{60}$	$\frac{9}{60}$	$\frac{10}{60}$	$\frac{5}{60}$	$\frac{6}{60}$
2	0.50	2	$\frac{11}{60}$	$\frac{9}{60}$	$\frac{10}{60}$	$\frac{5}{60}$	$\frac{6}{60}$
3	0.75	2	$\frac{11}{60}$	$\frac{9}{60}$	$\frac{10}{60}$	$\frac{5}{60}$	$\frac{6}{60}$
4	1.00	2	$\frac{11}{60}$	$\frac{9}{60}$	$\frac{10}{60}$	$\frac{5}{60}$	$\frac{6}{60}$
5	1.25	2	$\frac{11}{60}$	$\frac{9}{60}$	$\frac{10}{60}$	$\frac{5}{60}$	$\frac{6}{60}$
6	1.50	2	$\frac{11}{60}$	$\frac{9}{60}$	$\frac{10}{60}$	$\frac{5}{60}$	$\frac{6}{60}$
7	1.75	2	$\frac{11}{60}$	$\frac{9}{60}$	$\frac{10}{60}$	$\frac{5}{60}$	$\frac{6}{60}$
8	2.00	2	$\frac{11}{60}$	$\frac{9}{60}$	$\frac{10}{60}$	$\frac{5}{60}$	$\frac{6}{60}$
9	2.25	2	$\frac{11}{60}$	$\frac{9}{60}$	$\frac{10}{60}$	$\frac{5}{60}$	$\frac{6}{60}$
10	2.50	2	$\frac{11}{60}$	$\frac{9}{60}$	$\frac{10}{60}$	$\frac{5}{60}$	$\frac{6}{60}$

Table 4: Steady State Probabilities, p_j ; $j = 1, 2, \cdots, 22$ Corresponding to Breaking Down shares, $\tau_i = 1, 2$ and Minkwosi distance of a balanced system as discussed in (Agboola, 2016)

	Steady State Probabilities, $(p_1, p_2, \cdots, p_n)$ for States at Repair Completion for Case $(k_1 = k_2 = 2)$																						
Run	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}	p_{17}	p_{18}	p_{19}	p_{20}	p_{21}	p_{22}	**M.D**
1	***0.19428**	**0.13531**	**0.0741002**	**0.01883**	**0.17687**	**0.086054**	**0.040593**	**0.0031375**	**0.03916**	**0.0067456**	**0.0002766**	**0.009113**	**0.0030903**	**0.000708**	**0.035374**	**0.035374**	**0.000066**	**0.0009790**	****0.0000058**	**0.00006535**	**0.0000557**	**0.000013**	**0.23305**
2	***0.18722**	**0.128**	**0.04672**	**0.0622**	**0.144**	**0.082**	**0.0384**	**0.0104**	**0.032**	**0.0064**	**0.00091**	**0.018**	**0.0039**	**0.0047**	**0.0287**	**0.0287**	**0.00044**	**0.0016**	****0.000039**	**0.00017**	**0.00018**	**0.00004**	**0.17471**
3	***0.17915**	**0.1203**	**0.03372**	**0.0939**	**0.1151**	**0.0794**	**0.03621**	**0.0156**	**0.025478**	**0.00599**	**0.00138**	**0.02642**	**0.0043**	**0.0106**	**0.023**	**0.023**	**0.00101**	**0.0019**	**0.000089**	**0.00028**	**0.00033**	****0.000074**	**0.13081**
4	***0.17095**	**0.11469**	**0.022969**	**0.118401**	**0.09246**	**0.0758**	**0.03611**	**0.0167**	**0.023112**	**0.005988**	**0.00147**	**0.0345**	**0.00544**	**0.01518**	**0.0209**	**0.0209**	**0.00145**	**0.00203**	**0.000128**	**0.000485**	**0.0005398**	****0.000125**	**0.13014**
5	***0.16283**	**0.11076**	**0.01386**	**0.13729**	**0.07444**	**0.07221**	**0.03326**	**0.02288**	**0.01648**	**0.005511**	**0.00202**	**0.04197**	**0.00298**	**0.02605**	**0.01489**	**0.01489**	**0.002512**	**0.00206**	**0.0002214**	**0.0004937**	**0.000607**	****0.000134**	**0.05921**
6	***0.15491**	**0.10807**	**0.006244**	**0.147677**	**0.063998**	**0.068724**	**0.032468**	**0.02531**	**0.01417**	**0.005372**	**0.002231**	**0.04908**	**0.001677**	**0.03467**	**0.01280**	**0.01280**	**0.00337**	**0.0021254**	**0.0002971**	**0.0005342**	**0.0007574**	****0.000168**	**0.04091**
7	**0.147258**	**0.1063**	**0.005956**	***0.16305**	**0.05912**	**0.06536**	**0.03196**	**0.027175**	**0.01309**	**0.00528**	**0.002395**	**0.05577**	**0.001975**	**0.04355**	**0.011825**	**0.011825**	**0.004267**	**0.002291**	**0.0003761**	**0.000568**	**0.000961**	****0.000213**	**0.01695**

8	0.139924	0.1052	0.005664	* 0.17157	0.054325	0.062133	0.031769	0.028596	0.01203	0.00521364	0.0025206	0.06207	0.0020516	0.052534	0.010865	0.010865	0.005186	0.0024056	0.0004571	0.0007176	0.001163	** 0.000257	0
9	0.132926	0.1047	0.003196	* 0.17778	0.04966	0.05905	0.03152	0.02966	0.011138	0.005176	0.002615	0.067993	0.004055	0.061498	0.009932	0.009932	0.00611772	0.002474	0.0005392	0.000909717	0.0013569	** 0.000304	0.015796
10	0.1263	0.1045	0.0001429	* 0.17788	0.0452	0.0561	0.031499	0.02896	0.010	0.005157	0.002753	0.07357	0.006151	0.06693	0.009033	0.00903	0.006708	0.0025	0.000591	0.001533	0.00153675	** 0.00034	0.012133

In a subsequent paper we will exploit the given mix probabilities to obtain some performance measures as distribution of type A_1 and type A_2 machines under repairs or on repair queues, proportion of type A_1 and type A_2 in service, recurrence time of the system.

Table 5: Steady State Occupancy Probabilities, $p_j : j = 1, 2, \cdots, 7$ Corresponding to Breaking Down shares, $\tau_i = 1, 2$ and Minkwosi distance of a balanced system as discussed in (Agboola, 2016)

	Occupancy Probabilities at Repair Stations for Model I							
Run	q_1	q_2	q_3	q_4	q_5	q_6	q_7	**Murkowski Distance**
1	**0.6994**	**0.1689**	**0.00702**	**0.0880**	**0.03642**	**0.000121**	**0.0000182**	**0.21525**
2	**0.6679**	**0.1636**	**0.007299**	**0.1300**	**0.03077**	**0.000351**	**0.00008**	**0.16236**
3	**0.6422**	**0.1566**	**0.007373**	**0.1671**	**0.02594**	**0.000612**	**0.000163**	**0.11648**
4	**0.6194**	**0.1518**	**0.007461**	**0.1953**	**0.02464**	**0.001025**	**0.000248**	**0.08001**
5	**0.5992**	**0.1448**	**0.007528**	**0.2273**	**0.01946**	**0.001227**	**0.000356**	**0.04147**
6	**0.5913**	**0.1407**	**0.007603**	**0.2403**	**0.01830**	**0.001303**	**0.000465**	**0.02574**
7	**0.5888**	**0.1376**	**0.007671**	**0.2454**	**0.01838**	**0.001473**	**0.000589**	**0.01963**
8	**0.5767**	**0.1343**	**0.007734**	**0.2605**	**0.01846**	**0.0015803**	**0.000714**	**0.00000**
9	**0.5673**	**0.1314**	**0.007797**	**0.2718**	**0.01852**	**0.0022666**	**0.000843**	**0.01500**
10	**0.5640**	**0.1266**	**0.007810**	**0.2793**	**0.01824**	**0.003070**	**0.000931**	**0.24000**

4.0 Conclusion and discussion of result

Indicated in one (*) asterisk in Table 4 are the most frequent states for the runs, also indicated in two(**) asterisks are the less frequent states. The adopted parameters are justified for efficiency of service by the fact that the most frequent states are those for which the system is idle while the less frequent states are those for which the system has a high load for repairs. The last columns in Table 4 and Table 5 give the Minkwosi distance of the distribution of state probability and Minkwosi distance of the occupancy probabilities at repair station respectively, for each run compare to run 8 where there is some balanced in the breakdown rates ($\lambda_1 = \lambda_2$). It would appear that the closer the system is to a balanced situation, the less the discrepancy in the mix for the state probability distribution. We concluded that the relative mix can be rather different from each other depending on the appropriate parameter for the model.

5.0 References

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