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A steady poiseuille flow in the presence of a chemical reaction

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Abstract

This paper presents the solutions to the energy equation governing a steady poiseuille flow of a fluid in a channel with boundary conditions and in the presence of a chemical reaction using a numerical shooting technique that incorporates Runge – Kutta method of order 2. The results of this numerical solution are then presented in tables and graphs for different values of the various parameters.

An interesting situation observed is that the temperature in the channel flow decreases as the activation energy increases and also decreases as the fluid thermal conductivity increases.

Keywords: Steady Poiseuille flow; energy equations; boundary conditions; shooting technique; chemical reaction.

Introduction

Fluid involving chemical reactions and mass transfer have a wide range of applications. In fluid dynamics, many of these reactive flows have been generally modeled differently by including their appropriate chemical reactions in the Navier – Stokes equation. A.R.Bestman (1990) was the first to consider the mutual performance of the chemical reaction with Arrhenius activation energy for convective mass transfer in a vertical pipe immersed with porous media. He used perturbation method to obtain analytical solution. Kh.Abdul Maleque(2013) studied free convection MHD flow and heat with mass transfer over a porous vertical plate with binary chemical reaction and Arrhenius activation energy with heat generation/absorption and viscous dissipation.

M.Mustafa et al (2013) discussed the chemical reaction and activation energy with buoyancy effects on magneto-nanofluid passed through a vertical surface.

The purpose of this study is to use a numerical shooting technique to determine the effect of temperature on the activation energy and the thermal conductivity for a steady poiseuille flow in a channel with a given boundary condition.

Steady Poiseuille Flow in a channel

In this viscous channel flow, the two plates are stationary and placed a distance, H apart

Fig. 1

$U(0) = 0$ and $U(H) = 0$ imply the velocities at the lower and the upper plates are zeros respectively.

For incompressible flow, the continuity equation is given by;

$$\nabla . \vec{q} = 0 \qquad (1)$$

Where $\vec{q}$ is the velocity vector i.e $\vec{q} = (\vec{U}, \vec{V})$

Where $\vec{U}$ and $\vec{V}$ are the velocities of flow along the x and y components respectively.

Then, equation (1) becomes;

$$\frac{\partial \vec{U}}{\partial x} + \frac{\partial \vec{V}}{\partial y} = 0 \qquad (2)$$

The momentum equation for the channel flow is given by;

$$\rho \frac{\partial \vec{U}}{\partial t} = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 \vec{U}}{\partial y^2} \qquad (3)$$

where;

ρ is the density for a given fluid,

$\frac{\partial \vec{U}}{\partial t}$ is the rate of change of velocity $\vec{U}$ with time

$\frac{\partial P}{\partial x}$ is the pressure gradient along the x-component

$\frac{\partial \vec{U}}{\partial y}$ is the velocity gradient along the y-component

and μ is called the dynamic viscosity measured in kg/(ms)

Now, assuming there is no flow in the channel along the y-component, then equation (2) becomes;

$$\frac{\partial \vec{U}}{\partial x} = 0 \qquad (4)$$

For a Steady flow and a constant μ, then

$$\frac{\partial \vec{U}}{\partial t} = 0 \qquad (5)$$

Therefore, equation (3) becomes;

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 \vec{U}}{\partial y^2} \qquad (6)$$

Now, assume $\frac{\partial p}{\partial x} = A \ (where\ A\ is\ a\ constant)$

Then equation (6) becomes;

$$\frac{d^2\vec{U}}{dy^2} = \frac{A}{\mu} \tag{7}$$

Problem Formulation

We are considering the energy equation governing a steady Poiseuille flow of a fluid in a channel with boundary conditions and in the presence of a chemical reaction as;

$$\frac{\partial T}{\partial t} + \vec{U}\frac{\partial T}{\partial x} + \vec{V}\frac{\partial T}{\partial y} = \lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + Q_0 e^{-E/RT} \tag{8}$$

$$T(0) = T(H) = 2 \tag{9}$$

Where $\vec{U}$ = Velocity of the fluid along the x-component

$\vec{V}$ = Velocity of the fluid along the y-component

$\frac{\partial T}{\partial x}$ = Temperature gradient along the x-component

$\frac{\partial T}{\partial y}$ = Temperature gradient along the y-component

λ = Thermal Conductivity of the fluid in Watt per metre-kelvin ($W.m^{-1}.k^{-1}$)

Q_0= Exponential factor

E = Activation energy in J/mol

R = Universal molar gas constant= 8.3144$Jmol^{-1}k^{-1}$

T = Temperature in Kelvin

H = Distance apart between the two plates in the channel

$T(0)$ = Temperature at the Lower plate of the channel

$T(H)$ = Temperature at the Upper plate of the channel

Solution of the Problem

Considering the given problem in equation (8) with boundary conditions in equation (9), we assumed the following approximations;

$$\left.\begin{array}{c} \text{The flow is steady i. e. } \frac{\partial T}{\partial t} = 0 \\ \text{The flow is independent of x i. e } \frac{\partial T}{\partial x} = 0 \\ \vec{V} = 0 \end{array}\right\} \tag{10}$$

Putting equation (10) into equation (8), we obtain;

$$0 + 0 + 0 = \lambda\left(0 + \frac{\partial^2 T}{dy^2}\right) + Q_0 e^{-E/RT}$$

$$\lambda \frac{\partial^2 T}{\partial y^2} = -Q_0 e^{-E/RT}$$

$$\frac{d^2 T}{dy^2} = \frac{-Q_0}{\lambda} e^{-E/RT} \tag{11}$$

With $T(0) = T(H) = 2$

Using shooting method that incorporate Runge-kutta method of order 2. We let;

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} y \\ T \\ T' \end{pmatrix} = Y_i \tag{12}$$

$$\begin{pmatrix} \frac{\partial Y_1}{\partial y} \\ \frac{\partial Y_2}{\partial y} \\ \frac{\partial Y_3}{\partial y} \end{pmatrix} = \begin{pmatrix} Y_1' \\ Y_2' \\ Y_3' \end{pmatrix} = \begin{pmatrix} 1 \\ T' \\ T'' \end{pmatrix} = \begin{pmatrix} 1 \\ Y_3 \\ T'' \end{pmatrix} \tag{13}$$

From equation (11)

$$\frac{d^2 T}{dy^2} = \frac{-Q_0}{\lambda} e^{-E/RT}$$

This implies that;

$$T'' = \frac{-Q_0}{\lambda} e^{-E/RT} \tag{14}$$

Now, substituting for T'' in equation (13), we have;

$$\begin{pmatrix} Y_1' \\ Y_2' \\ Y_3' \end{pmatrix} = \begin{pmatrix} 1 \\ Y_3 \\ \frac{-Q_0}{\lambda} e^{-E/RT} \end{pmatrix} = f(Y_i) \tag{15}$$

Let $Q_0 = 1$, then equation (15) becomes;

$$\begin{pmatrix} Y_1' \\ Y_2' \\ Y_3' \end{pmatrix} = \begin{pmatrix} 1 \\ Y_3 \\ \frac{-1}{\lambda} e^{-E/_{RT}} \end{pmatrix} = f(Y_i) \quad (16)$$

$$\begin{pmatrix} Y_1(0) \\ Y_2(0) \\ Y_3(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ \beta \end{pmatrix} = Y(0) = Y_0 \quad (17)$$

Where β is guessed such that

$$T(0) = Y_2(0) = 2$$

Using Runge-kutta method of order 2:

$$Y_{i+1} = Y_i + 0.5(F_1 + F_2) \quad (18)$$

Where, $F_1 = hf(Y_i)$ *and* $F_2 = hf(Y_i + F_1)$ (19)

The above numerical schemes in equations (16), (17), (18) and (19) were used for the iterative solutions given $\beta = 0.5000/E = 0, \beta = 0.3071/E = 0.5$, $\beta = 0.1868/E = 1.0$ $all\ for\ \lambda = 1.0\ and\ \beta = 3.3829/\lambda = 0.1$, $\beta = 0.6219/\lambda = 0.5, and\ \beta = 0.3071/\lambda = 1.0$ $all\ for\ E = 0.5$

Table 1: Table of the guessed values of β against the activation energies, $E = 0, 0.5, 1.0\ when\ \lambda = 1.0$

Activation Energy $E(Jmol^{-1})$.	0	0.5	1.0
Guessed value of β	0.5000	0.3071	0.1868

Table 2: Table of the guessed value of β against the thermal conductivities, $\lambda = 0.1, 0.5, 1.0\ when\ E = 0.5$

Thermal Conductivity $\lambda\ (Wm^{-1}k^{-1})$	0.1	0.5	1.0
Guessed value of β	3.3829	0.6219	0.3071

With the numerical schemes in equations (16), (17) and (18), we obtained a table of temperature values for $E = 0$, $E = 0.5\ and\ E = 1.0\ when\ \lambda = 1.0$ for ten different iterations of y in the interval $0 \leq y \leq 1$ using a step length of $h = 0.1$

Table 3: Temperature profile when $\lambda = 1.0$

y	E=0	E=0.5	E=1.0
0	2	2	2
0.1	2.0450	2.0277	2.0168
0.2	2.0800	2.0492	2.0300
0.3	2.1050	2.0647	2.0394
0.4	2.1200	2.0739	2.0450
0.5	2.1250	2.0770	2.0450
0.6	2.1200	2.0739	2.0450
0.7	2.1050	2.0647	2.0394
0.8	2.0800	2.0492	2.0300
0.9	2.0450	2.0277	2.0168
1	2	2	2

Similarly, we obtained another table of temperature values for $\lambda = 0.1,\ \lambda = 0.5\ and\ \lambda = 1.0$ when $E = 0.5$ for same ten different iterations of y in the interval $0 \leq y \leq 1$ using same step length of $h = 0.1$

Table 4: Temperature Profile when $E = 0.5$

y	λ = 0.1	λ = 0.5	λ = 1.0
0	2	2	2
0.1	2.3080	2.0561	2.0277
0.2	2.5509	2.0999	2.0492
0.3	2.7261	2.1313	2.0647
0.4	2.8319	2.1502	2.0739
0.5	2.8672	2.1565	2.0770
0.6	2.8319	2.1502	2.0739
0.7	2.7261	2.1313	2.0647
0.8	2.5509	2.1000	2.0492
0.9	2.3080	2.0562	2.0277
1.0	2	2	2

Discussion of results and conclusion

Using the tables (3) and (4), graphs were plotted accordingly and the maximum values of the temperature (T) were obtained for the activation energies $E = 0, E = 0.5\ and\ E = 1.0$ to be $T = 2.1250, T = 2.0770\ and\ T = 2.0469$ respectively and the thermal conductivities $\lambda = 0.1,\ \lambda = 0.5\ and\ \lambda = 1.0$ to be $T = 2.8672,\ T = 2.1565\ and\ T = 2.0770$ respectively.

Fig(2): The graph of Temperature (in kelvin) against positions y for some values of activation energies.

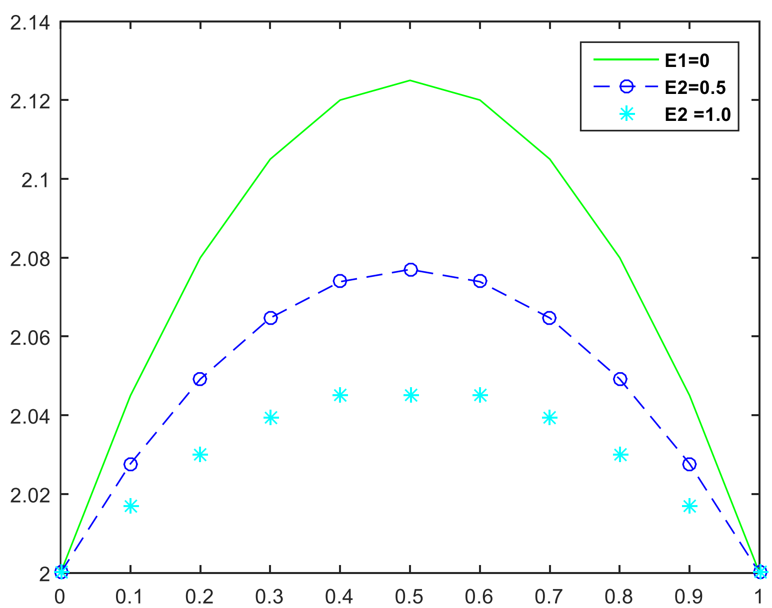
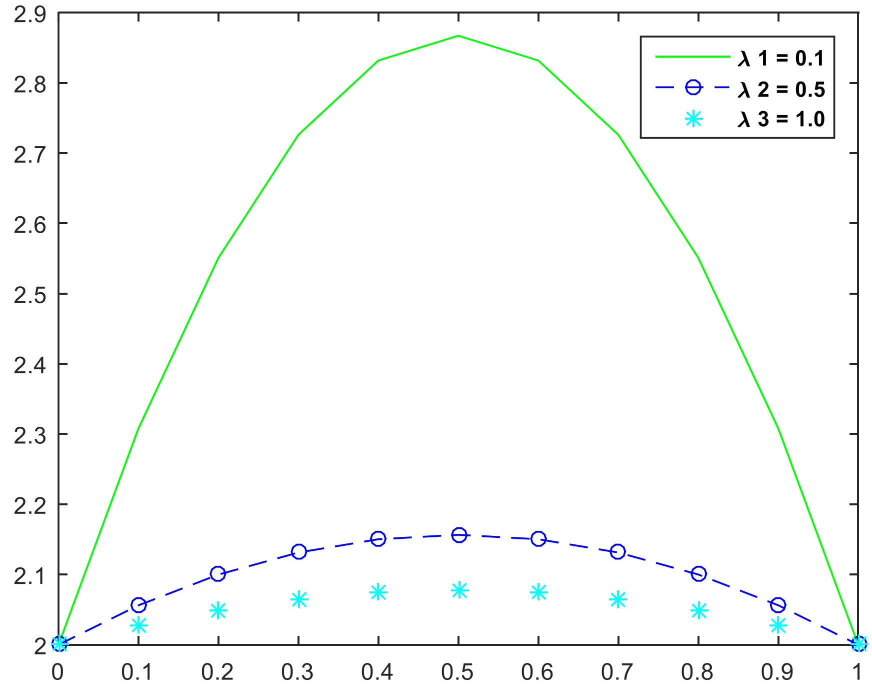


Fig (3): The graph of temperature (in kelvin) against positions y for various values of thermal conductivities.



The results from the graphs showed that the temperature in the channel flow decreases as the activation energy increases and also decreases as the thermal conductivity of the fluid increases.

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