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Research

On the definition of Multiplier

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Abstract

In this study R-Multiplier is defined as an optimizing function on chosen mathematical structures.With the continuity condition satisfied, R-Multiplier attains a supremum or an infimum at some point of R.

Keywords: *Multiplier, optimizing function, torus, lattice, langrange*

Introduction

In the arithmetic sense, a multiplier is a number that multiplies another (Hollands, 1981). Algebra wise, a multiplier is a commutator: if G is a group, the commutator of two elements a, b $\in$ G is denoted [a,b] and defined by $[a,b]= a^{-1} b^{-1} ab$. Thus in an albelian group G, the commutator of a, b $\in$ G is the identity element.

A multiplier can also be used as a Langrange multiplier (Langrange, 1906). This is an optimizing constant to be determined. If the minimum or maximum value of the function f(x,y) (1.1)
is desired, subject to some constraints

$$g(x,y) = 0 \quad (1.2)$$

we determine first, the stationary points of f(x,y). Subsequently, we determine whether the stationary points are minimum or maximum points of the function f(x,y), subject to the constrain g(x,y) = 0.

Kleppner (1965) used a multiplier as a function on a topological subsemigroup, S of the reals with values in the Torus denoted by T:

$$\sigma: S \times S \rightarrow T \quad (1.1.1)$$

$$\ni \sigma(xy,z)\sigma(x,y) = \sigma(x,y)\sigma(y,z) \quad (1.1.2)$$

In another setting, Kleppner (1993) defined a multiplier as a function:

$$\omega: G \times G \rightarrow T \quad (1.1.3)$$

$$\ni \omega(xy,z)\omega(x,y) = \omega(x,yz)\omega(y,z) \quad (1.1.4)$$

and

$$\omega(e,x) = \omega(x,e) = 1 \quad (1.1.5)$$

Where G is a locally compact group

Varadarajam (1970) defined K-multiplier as a function

$$\omega: G \times G \rightarrow K \quad (1.1.6)$$

$$\ni \omega(xy,z)\omega(x,y) = \omega(x,yz)\omega(y,z) \quad (1.1.7)$$

and

$$\omega(x,e) = \omega(e,x) = 1 \quad (1.1.8)$$

where G and K are lcsc (locally compact groups satisfying the second axiom of countability) with K abelian (Varadarajan, 1970). Adelodun (2002) introduced a real valued multiplier. Thus, paving way for using a multiplier as an optimizing function. In this paper, we consider a multiplier as a function.

Multipliers

H-Multipliers

Let G, H be lcsc (locally compact groups satisfying the second axiom of countability) with H abelian, then by an H-Multiplier for G we mean a function:

$$\gamma: GxG \rightarrow H$$

such that:

$$\gamma(xy,z)\gamma(x,y) = \gamma(x,yz)\gamma(y,z) \quad (2.1)$$

$$\forall x,y,z \in G$$

and

γ (x,e) = γ (e,x)= e_H

with x $\in$ G, e_H is the identity element in H

Remark: If as usual we define a multiplier in T (=Torus), we omit the qualifier before the multiplier, that is, if

$$\gamma: GxG \to T \quad (2.2)$$

We do not write a T-multiplier, we simply write a multiplier.

R-Multiplier

If we define ω on G (G is any ring) as a function to R

$$\omega: G \times G \to R \quad (2.3)$$

such that:

$$\omega(x+y,z)+\omega(x,y)=\omega(x,y+z)+\omega(y,z)$$

$$\forall x,y,z \in G \quad (2.4)$$

and

$$\omega(0,x)=\omega(0,x)=0 \qquad \forall x \in G, \quad (2.5)$$

we have an R-Multiplier

Here both G and ω are written additively. However, if we also write G and ω multiplicatively:

$$\omega: G \times G \to \quad (2.6)$$

such that

ω (x y,z) ω (x,y)= ω (x, y z) ω (y,z) (2.7)

ω is still an R-Multiplier

Results

Illustration of The Definition

The definition of H-multiplier or R-Multiplier is given as an abstracion. In this section, we look for operation that satisfies the definition.

Algeraic Operation

If we define ω (x,y) = xy, then (2.7) is satisfied. We will examine if (2.5) is also satisfied

$$\omega(x+y,z)+\omega(x,y)=\omega(x,y+z)+\omega(y,z)$$

I. h. s.

$$\omega(x+y,z)+\omega(x,y)=(x+y)z+xy:$$

= xy+yz+xy (3.1)

G is a ring where multiplication is distributive over addition.

r. h. s.

$$\omega(x,y+z)+\omega(x,y)=x(y+z)+yz$$

= xy+xz+yz (3.2)

(G is where multiplication is distributive over addition)

Comparing (3.1) and (3.2) we have the equality. Hence, ω is an R-Multiplier. If we write both G and ω multiplicatively with z=x, then from (1.1.7)

$$\omega(xy,z)\omega(x,y)=\omega(x,yz)\omega(y,z)$$

so that

xyxxy=xyxyx $\Rightarrow$ $x^3y^2=x^3y^2$ (3.3)

satisfy the equality.

Theorem

Let U(R) denote the set of all units in a ring R (Kuku, 1980) and consider the function

$$\sigma: U(R) \times U(R) \to R$$

Then σ is such that

(i) $\sigma(xy,z)=\sigma(x,y)=\sigma(x,yz)\sigma(y,z)$

(ii) $\sigma(x,y)=\sigma(y,x)=1$ (3.4)

For all units elements (x,y,z) $\in$ U(R)

That is, σ is an R-Multiplier on U(R)

Proof

If we define σ (x,y) = xy and since all elements of U(R) are units $\sigma(\times, y)=\sigma(y,\times)=1$

Satisfying (ii). For: condition (i)

$$\sigma(xy,z)\sigma(x,y)=\sigma(x,yz)\sigma(y,z)$$

becomes

$$\sigma(1,z)\sigma(x,y)=\sigma(x,1)\sigma(y,z),$$

all elements are units, that is,

σ (x,y) = σ (y,z) by (ii) of conditions under the theorem with the definition of σ given

σ (x,y) = σ (y, z) in U(R)

Hence the proof

Next section is an example of where a suitably defined multiplier can be used to determine the least subset in a class of subsets. That is σ can be used as minimizing function.

The set Theoretic intersection, $\cap$

Let P(R) denote a power set of R and that (P(R), $\cap$)

is a locally compact group and define

σ :P(R) x P(R) $\rightarrow$ R

by σ (X,Y)= X $\cap$ Y $\forall X, Y \in P$ (R) (3.5)

where X $\subset Y \subset Z$, is ordered by set inclusion.

Then σ :P(R) x P(R) $\rightarrow$ R

is such that

$$\sigma(X \cap Y, Z) \cap \sigma(X, Y) = \sigma(X, Y \cap Z) \cap \sigma(Y, Z) \quad (3.6)$$

and

$\sigma(X, \varnothing) = \sigma(\varnothing, X) = \varnothing$ by definition

where $\varnothing$ is the null subset in P(R)

Now,

$$\sigma(X \cap Y, Z) \cap \sigma(X, Y)$$

$$= \sigma(X, Y \cap Z) \cap \sigma(Y, Z)$$

I.h.s

$$\sigma(X \cap Y, Z)\sigma(X, Y) = \sigma(X, Z) \cap \sigma(X, Y)$$

$$= X \cap X = X$$

r.h.s

$$\sigma(X, Y \cap Z) \cap \sigma(Y, Z) = \sigma(X, Y) \cap \sigma(Y, Z)$$

$$= (X \cap Y) \cap (Y \cap Z)$$

$$= X \cap Y = X$$

Hence (3.6) is satisfied and σ is a multiplier

Similarly, the function

σ : P(R) x P(R) $\rightarrow$ R

such that

$$\sigma(X \cup Y, Z) \cup \sigma(X, Y) = \sigma(X, Y \cup Z) \cup \sigma(Y, Z) \quad (3.7)$$

where $\sigma(X, Y) = X \cup Y$

shows that the multipliers, σ is a maximizing function. Here, (P(R) $\cup$) is a locally compact subsemigroup. So, (3.6) is satisfied.

“meet” in a Lattice.

Let L $\subseteq$ R be a lattice with partial orders. $x \leq y \leq z$ on L

where each pair x and y has a greatest lower bound (g.l.b) denoted by $x \wedge y = x$

Define

$\beta: LxL \rightarrow$ R

such that

$$\beta(x \wedge y, z) \wedge \beta(x, y) = \beta(x, y \wedge z) \wedge \beta(y, z) \quad (3.9)$$

I.h.s

$$\beta(x \wedge y, z) \wedge \beta(x, y)$$

$$= \beta(x, z) \wedge \beta(x, y)$$

$$= x \wedge x$$

$$= x \quad (3.10)$$

r.h.s

$$\beta(x, y \wedge z) \wedge \beta(y, z)$$

$$= \beta(x, y) \wedge \beta(y, z)$$

$$= (x \wedge y)$$

$$= x$$

So, β is a multiplier since the equality of (3.10) holds, from (3.5) and (3.11)

Conclusion

The concept of R-Multiplier is hereby introduced. A careful investigation of the concept led to making a multiplier an optimization function. Further investigation regarding the concept is in progress.

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