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A proof of continuity of $\mathbb{R}$-Multiplier

*Adelodun, Joe F.

Babcock University, Ilishan-Remo, Department of Mathematics and Computer Science,
PMB 21244, Ikeja, Lagos, Nigeria

*Corresponding author <joeadelodun@yahoo.com>

Abstract

This paper, gives a proof of the continuity of $\mathbb{R}$-Multiplier, using the open ball definition of continuity. Thus, $\mathbb{R}$-Multiplier satisfies optimization criterion for mapping.

Keywords: Constructive method, continuity, proof, R -Multiplier

Introduction

Adelodun (2002) highlighted the different meaning and usage of a multiplier which includes its use as a mapping or a function.

Let G be a locally compact group and T the torus. Kleppner (1965) defined a multiplier as a mapping:

$\omega: G \times G \rightarrow T$

such that

$\omega(xy,z)\ \omega(x,y) = \omega(x,yz)\omega(y,z)$ (1)

$\omega(x,e) = \omega(e,x) = 1$ (2)

Consider a topological subsemigroup, S, of $\mathbb{R}$, In another setting, Kleppner (1993) defined a multiplier as a function:

$\sigma: S \times S \rightarrow T$

such that

$\sigma(xy, z)\ \sigma(x,y) = \sigma(x,yz)\Phi(y,z)$ (3)

If K denotes a compact abelian subgroup of a locally compact group G, Varadarajan (1970) defined a K-Multiplier as a mapping:

$M;\ G \times G \rightarrow K$

such that

$M(xy,z)M(x,y) = M(x,yz)M(y,z)$ (4)

$M(x,e) = M(e,x) = 1$ (5)

Here, the concept of an R-Multiplier is introduced. The goal is to show that an R-Multiplier satisfies the optimization criterion, so that we can relate an $\mathbb{R}$-Multiplier to an optimization problem.

Definition

Let G be compact topological group. Then the mapping

$\omega: G \times G \rightarrow \mathbb{R}$

such that

$\omega(xy,z)\omega(x,y) = \omega(x,yz)\ \omega(y,z)$

$\omega(x,e) = \omega(e,x) = 1$

is called an $\mathbb{R}$-Multiplier.

An Optimization Criterion.

Bamigbola and Adelodun (2002) have shown that an optimization criterion may be stated in terms of a theorem as follows:

Theorem 2.1

A continuous mapping

$\sigma: X \rightarrow \mathbb{R}$

of a compact metric space, X into $\mathbb{R}$ assumes a minimum or maximum at some point of X. As consequence of the criterion, a continuous mapping may be defined on a compact topological group, in the following theorem:

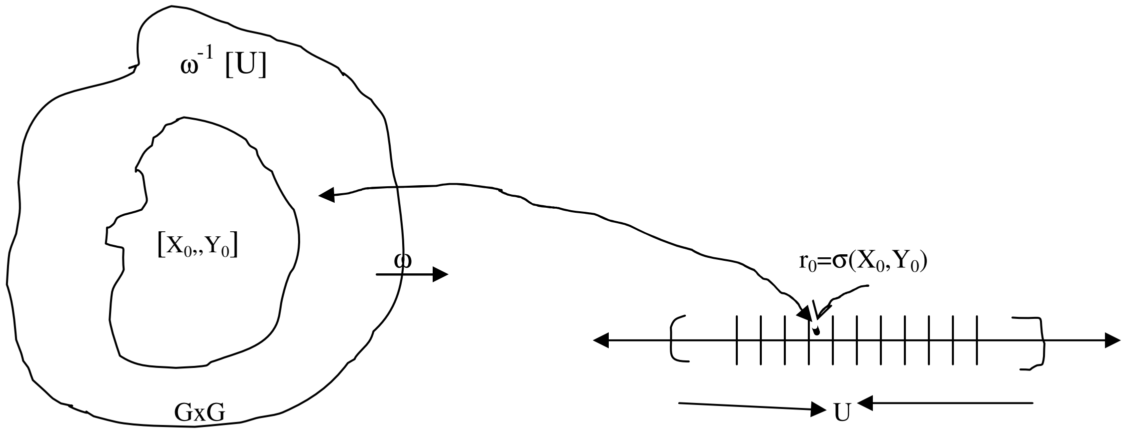

Fig. 1………………..

Theorem 2.2
Let G be a compact topological group. Then a continuous mapping

$\sigma: G \to \mathbb{R}$

of G into R assumes a minimum or a maximum at some point of G.

If the ℝ-Multiplier σ is continuous, then σ becomes an optimization mapping. We state an optimization problem in terms of an ℝ-Multiplier as a theorem, which is given and proved in the next section.

The Main Theorem

Theorem 3.1
Let G be a compact topological group. Suppose that G x G caries a topology defined by metric $\rho_{G \times G}$. Then an R-Multiplier

$\omega: G \times G \to \mathbb{R}$

assumes a minimum or a maximum at some point of G if ω is continuous.

Remark: The criterion is that ω must be continuous. A proof of the continuity of ω, using open-ball definition of continuity is given. To do this we require the following lemma

Lemma 3.2 Let G be a compact topological group. Suppose the topology on G x G is defined by metric. The mapping

$\sigma: G \times G \to \mathbb{R}$

is continuous at (x_0,y_0), $\in$ G x G if for each open ball, $B_{\mathbb{R}}[\sigma(x_0, y_0),\varepsilon]$, there exists an open ball $B_{GxG}\{(x_0,y_0),\delta\}$
such that

$\sigma[B_{GxG}(x_0,y_0), \delta] \subset B_{\mathbb{R}}[\sigma(x_0,y_0),\varepsilon]$

Proof

The intention is to prove that

$\sigma: G \times G \to \mathbb{R}$

is continuous at $(x_0,y_0) \in GxG$ if and for each open ball $B_{\mathbb{R}}[\sigma(x_0,y_0), \varepsilon]$ in $\mathbb{R}$, there exists open ball $B_{GxG}\{(x_0,y_0),\delta\}$ such that

$\sigma[B_{GxG}\{(x_0,y_0),\delta\}] \subset B_{\mathbb{R}}[\sigma(x_0,y_0),\varepsilon]$.

Assume σ is continuous at (x_0,y_0), σ is continuous at (x_0,y_0)]$\Leftrightarrow$Given $\varepsilon > 0$, $\exists\ \delta > 0$ such that

$\rho_{GxG}[((x_0,y_0), (x,y,)] < \delta \Rightarrow B_{\mathbb{R}}[\sigma(x_0,y_0),(x,y)] < \varepsilon$
$\Leftrightarrow$Given $\in > 0$, $\exists\ \delta > 0$ such that

$(x,y) \in B_{GxG}\ \{(x_0,y_0), \delta\} \Rightarrow \sigma(x, y) \in B_{\mathbb{R}}[\sigma(x_0,y_0), \varepsilon]$
$\Leftrightarrow$Given $\varepsilon > 0$, $\exists$, $\delta > 0$ such that

$$\sigma[B_{GxG}(x_0,y_0), \delta] \subset B_{\mathbb{R}}[\sigma(x_0,y_0), \varepsilon]$$

since (x_0,y_0) is arbitrary, σ is continuous.

Proof of Theorem 3.1
The mapping

$\omega: GxG \to \mathbb{R}$

such that

$\omega(xy,z)\omega(x,y)= \omega(x,yz)\omega(y,z)$

$\omega(x,e) = \omega(e,x)=1$

is continuous iff $\omega^{-1}(U)$ is an open set in GxG for each open set $U \subset \mathbb{R}$

We illustrate the theorem as in Fig. 1.

It is required to prove that σ is continuous if for each open set $U \subset R$, $\omega^{-1}(U)$ is an open set in GxG.

__**If Part**__
Suppose

$\omega: GxG \to \mathbb{R}$

such that

$\omega(xy,z)\ \omega(x,y)= \omega(x,yz)\ \omega(y,z)$

$\omega(x,e)= \omega(e,x)=1$

is continuous. We take any open set U ⊂ |. We shall show that ω^{-1}(U) is an open set in GxG.
Our method of approach is to show continuity of ω at an arbitrary point, (x_0,y_0).
Let $(x_0,y_0) \in \omega^{-1}(U)$, then $\omega(x_0,y_0) \in U$, U open in |. Therefore, given ε >0 such that
B |[ω((x_0,y_0), ε] . But ω is continuous.
So, by lemma, 3.2 ∃ δ > 0 such that
ω[B_{GxG} (x_0,y_0), δ] ⊂ B |[ω(x_0,y_0), ε]. ⊂ U
i.e ω[B_{GxG}(x_0,y_0),δ] ⊂ U.
i.e ω[B_{GxG}(x_0,y_0),δ ω^{-1} (U).
Thus for an arbitrary point (x0,y0) ω^{-1}(U), we find an open ball centered at (x_0,y_0) which is contained in ω^{-1}(U). Hence ω^{-1}(U) is open in G x G.

Only if part

Suppose that for any open set U δ |, ω^{-1}(U) is open in G x G. We prove that T is continuous. To this end, let $(x_0,y_0) \in$ GxG be arbitrary, then it is sufficient to prove that ω is continuous at (x_0,y_0). Take ε > 0, then B |[ω(x_0,y_0), ε] is an open set in R. By hypothesis,
ω^{-1}[B |(ω (x_0,y_0) ε] is open set in G x G.
Moreover, $(x_0,y_0) \in \omega^{-1}$[B |{ ω($x_0,y_0$), ε}].
Hence, ω[B_{GxG}{(x_0,y_0),δ}]⊂ B |[T(x_0,y_0), ε].
So, ω is continuous at (x_0,y_0) and hence, ω is continuous, since (x_0,y_0) is arbitrary.

From the continuity of ω at (x_0,y_0), it follows that ω is continuous at (x_0,y_0,z_0) (x_0,y_0) since the product of continuous function is continuous.
Hence, ω(xy,z) ω(x,y)= ω(x,yz) ω (y,z)is continuous.
Since, ω(x,e) = ω(e,x) = 1 is a point, then
ω(x,e) = ω(e,x) = 1 is continuous. Hence, an |-multiplier
ω: G x G → | is continuous.

Conclusion

Since ω as an |-Multiplier on a compact topological space is continuous, it assumes a minimum and a maximum at some point by the optimization criterion. We can therefore regard a |-Multiplier as an optimization problem and vice-versa.

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