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The resonance method of determining the dielectric constant of a specimen (A4 Paper)

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Abstract

An LC based resonance circuit consisting of precision electrodes was used to determine the dielectric constant of A4 paper. The resonance of the circuit (with air as the dielectric) was first determined by continuously adjusting the fine and coarse control knobs of the variable capacitor of the LC tank. The half power bandwidth bo of the resonance curve was found by adjusting the fine control. The A4 specimen was then inserted appropriately. A new resonance curve with new halfpower bandwidth was obtained. Harrop (1972) showed that the specimens' capacitance is a function of the change in variable capacitance needed to achieve resonance. This was then used to obtain the dielectric constant.

The dielectric constant of the A4 paper specimen was obtained as 1.203, which was close to the expected value when compared with standard values obtained from published literature. The resonance method is quite flexible and may be used to determine the dielectric constant of industrial materials such as lubricating oils, transformer oils, egusi oil and other paper types.

Keywords: *Dielectric Constant of A4 Paper*

Introduction

Elwell et al. (1978) and Grant (1990) defined a dielectric as an insulating material, which exhibits electric dipole polarization, in which an electrostatic field can exist under the influence of an electric field. Such substances are capable of sustaining electrical stress. Simple (non-polar) dielectrics include Glass, Mica, Plastics, Paper, Wax, Gasses, Polystyrene etc. In the atomic structure of a simple (non-polar) dielectric, molecules are closely packed together that each other's short-range field are interactive. Grant (1990) showed that because the average field E_{local} acting on individual molecules is not the same as the macroscopic field E, simple (non-polar) dielectrics have no permanent dipole moment. Polar dielectrics exhibit sharing of electrons between atoms, which make them almost ionic in character and have permanent dipole moment. In anisotropic dielectrics the magnitude and direction of the induced dipole moment depends on the orientation of the crystal axes with respect to the field direction. While isotropic dielectrics imitate an electric field whatever their orientation.

Duffin (1965) broadly classifies dielectrics into *homogeneous* and *ferroelectrics* dielectrics. A dielectric is homogenous if its electric susceptibility χ_e is independent of position. Ferroelectrics are dielectrics that exhibit polarization even in the absence of an applied electric field. Ferroelectrics are nonlinear dielectrics. Apart from the dependence of permittivity on electric field intensity; the most essential features of this class of dielectrics include hysteresis, very high values of permittivity and presence of a spontaneous polarization without an external electric field.

This research aims at providing an alternative means of obtaining replacements for materials whose dielectric constant is a priori especially in designing components such as capacitors where the value of the dielectric constant of the material is of significance to the realizable value of the capacitance of the capacitor under design.

Dielectric Constant

Absolute permittivity ε is a term used to define dielectric constant. Valerie (1990) defined this to be the degree to which a medium can resist the flow of charge and may be expressed as a ratio of the electric displacement to the intensity of the electric field that produces it. As applied to free space ε transforms to ε_o and is called *electric constant*. The ratio of the absolute permittivity of a medium to the electric constant is called *relative permittivity*. The value varies from unity to ten for ordinary materials and over 4000 for ferroelectrics (Valerie, 1990). This is also called the *dielectric constant*. For a capacitor this is expressed as the ratio of the capacitance of the capacitor to the capacitance it would posses if the dielectric were removed.

The *dielectric constant* may be used in the following ways;

i. As Liquid Level Control for both hazardous and non hazardous materials.
ii. For determining the water or moisture content of materials.
iii. Detecting wire cuts for wire sizes down to 0.003".
iv. Detecting plastic pellet level in a hopper moulding processes.
v. Detecting small metal parts in inductive sensors.
vi. For modelling human tissue in developing dielectric managing system for the kidney and level organs, without possible corrections in the bovine dielectric permittivity. (Agba et al, 2002).

This research is aimed at measuring the dielectric constant of A4 paper. The methodology used is based on electronic resonance method and it involves the fabrication of a suitable coil for use in a locally constructed resonator. The circuit was connected to a 20MHz dual input oscilloscope to monitor the signal input, output, amplitude and frequency.

Theoretical formulation of some properties of dielectrics

Polarisation: Consider an electric field E generated between the parallel plates of a capacitor with a dielectric placed between the plates as shown in Fig. 1. Under the influence of the electric field, the molecules of the dielectric become polarized (separating +q and –q by a distance r determined by the electric field intensity) and will acquire electric dipole moment P. And since polarization is dependent on the electric field it may be expressed in terms of the susceptibility χ as (Elwell et al 1978; Tareev 1975).

$$P = \chi E \qquad (1)$$

Relative Permittivity: Assuming a parallel plate capacitor with area A and separated by a distance d is placed in a vacuum. Surface charges ±σ appear on the plates. The field E_o on the surface S due to charge ±σ is given as (Elwell et al 1978; Tareev 1975).

$$E_o = \sigma / \varepsilon_o$$

$$C_o = \frac{Charge}{Potential\ difference} = \frac{A\sigma}{E_o d} = \frac{\varepsilon_o A}{d} \qquad (2)$$

C_o represents the capacitance developed between the plates in a vacuum and σA the flux out of S as given by Gauss law.

(a) In a Vacuum (b) filled with dielectric materials.

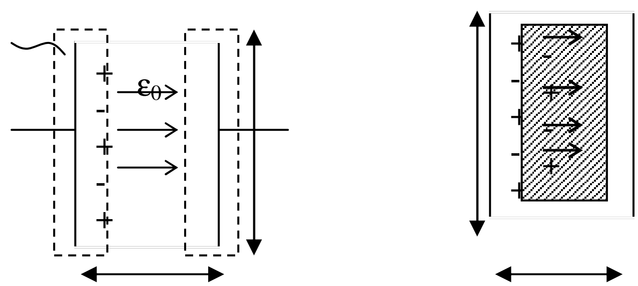


Fig 1 Parallel Plate Capacitor

The ratio of the capacitance with the dielectric medium in place, C_m to that in a vacuum C_o is given as (Duffin 1965; Grant 1990).

$$\frac{C_m}{C_o} = \frac{\varepsilon_m}{\varepsilon_o} = \varepsilon_r \qquad (3)$$

ε_m is regarded as the permittivity of the dielectric material and ε_r the relative permittivity or the dielectric constant of the material.

Conductivity: The Conductance G (Ω^{-1} or Siemens) between the plates is given by (Harrop 1972; Tareev, 1975).

$$G = \frac{\sigma` A}{d} = \frac{1}{R} \qquad (4)$$

where σ` is the conductivity of the dielectric material. Neglecting edge effects, the capacitance is related to a parallel plate configuration as (Harrop 1972)

$$C = \frac{\varepsilon` \varepsilon_o A}{d} = \frac{\varepsilon` \varepsilon_o G}{\sigma`} \qquad (5)$$

ε` is the relative permittivity of the dielectric.

Loss Tangent and the Q-Factor: Using complex notation, the impedance, Z, of a dielectric is given by (Harrop, 1972)

$$\frac{1}{Z} = \frac{1}{R} + jwC \qquad (6)$$

j is $\sqrt{-1}$, w is 2πf, R the associated resistance and C the capacitance. We define a generalized permittivity that has both real and imaginary parts and includes both resistive and capacitive contributions befitting to equation (6) as

$$\varepsilon^* = \varepsilon` - j\varepsilon`` \quad (7a)$$

$$\text{And} \quad C^* = \varepsilon^* C_o \quad (7b)$$

$j\varepsilon``$ is associated with the resistive vector and C* the complex capacitance of a slab of dielectric. If there are dielectric losses, the permittivity is expressed with an in-phase and an out-phase component. The negative sign of equation (7a) implies the dielectric absorbs power rather than generate.

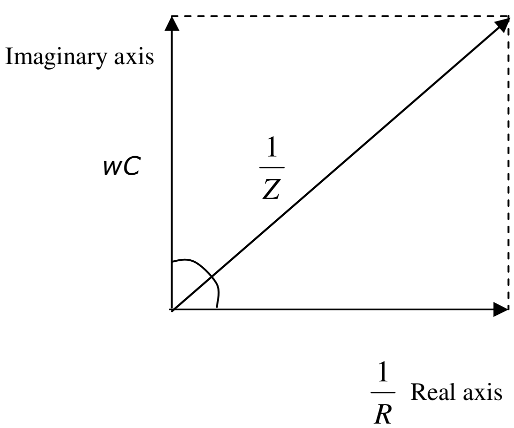


Fig. 2 Vector diagram showing impedance in a dielectric

When a potential V is applied across a parallel plate capacitor, the energy stored is $w = \frac{1}{2}CV^2$. Recall that V = Ed, where E is the applied field, noting that $C = \frac{\varepsilon A}{d}$, then $w = \frac{1}{2}\varepsilon E^2 \tau$, where τ = Ad (the volume and is 1 for a unit volume) and ε the permittivity of a medium ordinarily expressed which translates to ε_m for a given medium. With potential V applied, a current $I = jwCV$ flows into the resonator so that the impedance is given as (Harrop 1972)

$$Z = \frac{1}{jwC^*} = \frac{1}{jwC_o(\varepsilon` - j\varepsilon``)} \quad (8)$$

Combining equations (6), (7) and (8) and equating the real and imaginary parts, one observes

$$\varepsilon` = \frac{C}{C_o} \quad and \quad \varepsilon`` = \frac{1}{wC_oR} = \frac{\sigma`}{w\varepsilon_o}$$

that where ε` is same as ε_r. It is important to note here that $\frac{1}{wRC} = \frac{\varepsilon``}{\varepsilon`} = \tan\delta$. Tan δ is referred to as the dissipation loss or the loss tangent and is consistent with $\delta = 90^o - \theta$ shown in Fig 2. The reciprocal of the loss tangent gives the quality factor Q.

$$Q = \frac{1}{\tan\delta} = \cot\delta = \tan\theta$$

Energy Storage and Power factor:
When an electric field is applied to a dielectric the displacement of the electric charges will be a function of the energy stored. When a potential V is applied to the capacitor, energy is stored. If the potential is allowed to vary at an angular frequency w, Harrop (1972) gave the corresponding power factor as;

$$P = \tfrac{1}{2}\,\omega\varepsilon E^2_o \tan\delta \quad (9)$$

Radiation Properties of Dielectrics:
Visible light rays alone exert a definite effect on dielectrics and semiconductors and increase their conductivity under illumination. A prolong action of the intensive flux of light rays speeds up the ageing of a number of organic insulating materials. Thus Petroleum oil, Rubber and Capron display a low light resistance. Under ultraviolet rays and ionizing radiation materials lose some of their mechanical strength and elasticity (Harrop 1972; Tareev, 1975).

Resonance method measuring dielectric constant
At low frequencies of 10^{-4} to 10^{-1}Hz, the step response method, which is based on the fact that the ac loss frequency spectra are the Fourier transforms of the dc current with time, is more suitable. This involves complex numerical integrations. Hamon (2000) derived an approximation given as

$$\varepsilon`` \approx \frac{i}{wCV} \quad (10)$$

where ε`` is the imaginary part of the complex permittivity at frequency w = 2πf and i is the charging current.

At audio frequencies of 10^0 to 10^5Hz, the Schering bridge method, which is based on adjusting the capacitance and resistance to balance the current in a sample, is quiet suitable.

At radio frequencies of 10^4 to 10^9Hz, the resonance method, based on the use of precision electrodes with minimal leads is more suitable. This is a modified version of the Hartshorn and Ward technique. In this method, resonance is first detected with air in the sample position by adjusting the variable capacitor, which consists of a fine and coarse control. The half power bandwidth b_o of the resonance curve is found by adjusting the fine control. The specimen of material whose dielectric constant is to be measured is then inserted appropriately. A new resonance curve with new halfpower bandwidth b_i is obtained. The capacitance of the specimen is given by the change in variable capacitance needed to achieve resonance again. ε` is found from (Maddock and Calcutt, 1995; Harrop 1972)

$$C = \varepsilon` \varepsilon_o \frac{A}{d} \quad (11)$$

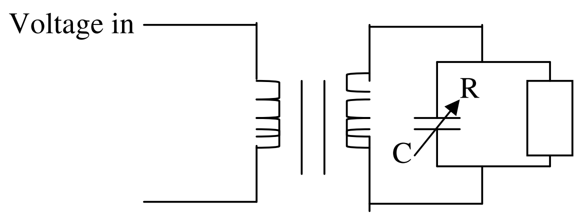


Fig 3 Hartshorn and Ward apparatus

Tan δ, the dissipation factor also called the attenuation factor is given as (Maddock and Calcutt, 1995)

$$\tan\delta = \frac{b_i - b_o}{2C} \quad (12)$$

The capacitance cell

The capacitor plates, which make up the sample cell, was constructed at the Jos University Equipment Maintenance Workshop. The flat headed drilling screw for the terminal of the electrode was improvised with the aperture slightly curved inward. The inductor coil was locally wound.

The sample cell used consisted of two parallel brass circular electrodes of average diameter of 25mm, which was mounted at the two ends of a Perspex cylinder of diameter 26mm. The electrodes have average thickness of approximately 10mm. Inter-electrode distance (d) within the Perspex cylinder is approximately 1mm.

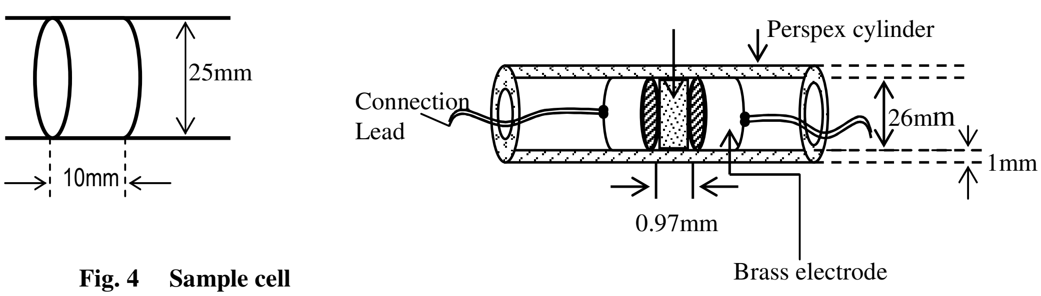


Fig. 4 Sample cell

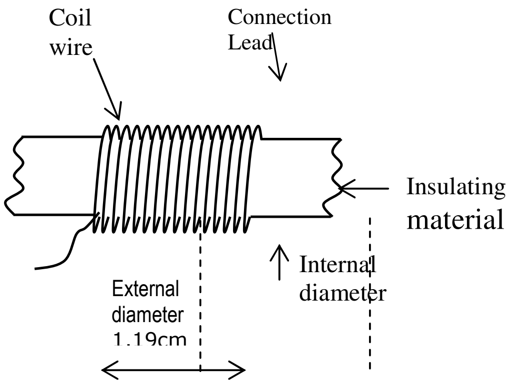


Fig. 5 Inductor coil wound around insulating material

Inserting the constants $\varepsilon_o = 8.854 \times 10^{-12}$ Fm^{-1}, and $d = 5 \times 10^{-3}$m into equation (2), the area of the Brass Electrode can be computed from, $A = \pi r^2$ since the radius of the brass electrode was measured as 12.5×10^{-3}m. Substituting values gives $C = 8.7 \times 10^{-13}$ F.

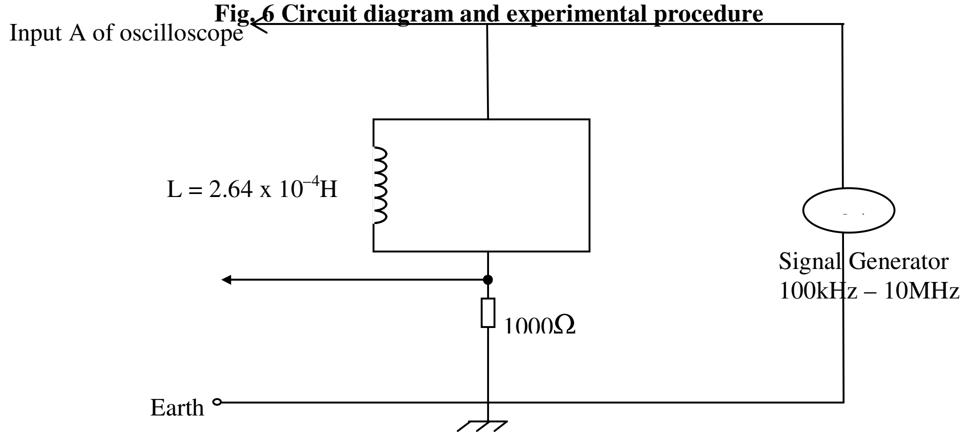
THE INDUCTANCE COIL

The enamel insulated copper wire was wound round cylindrical insulating material or core, making a total of 180 turns. The cylindrical insulating material has the following dimensions. The external and internal diameters are approximately 1.19×10^{-2}m and 0.99×10^{-2}m respectively. The length is 1.4×10^{-2}m. The inductance of the coil is given as (Harrop, 1972)

$$L = \frac{\mu_o N^2 A}{\ell} \qquad (13)$$

where $\mu_o = 4\pi \times 10^{-7}$ Hm^{-1}, N is number of turns (180), ℓ is the length of coil (1.4×10^{-2}m) and A the area occupied by the coil, which translates to the diameter of the coil = $2.15/2\ cm$ and radius r = $2.15/4$ = 5.38×10^{-3}m. A= πr^2 ($9.09 \times 10^{-5} m^2$). Substituting values, we obtain L as 2.64×10^{-4}H.

Fig. 6 Circuit diagram and experimental procedure



Resonance circuit for the

determinationof the dielectric constant of paper. The circuit was connected to the oscilloscope as shown in Fig. 6. The dielectric constant was first determined with air in the position of the sample cell. Then the dielectric constant was again determined with A4 paper was trimmed to fit in between the capacitor plates acting as the dielectric in the sample cell. Signals corresponding to those presented on table 1 were fed into the circuit from the signal generator and the peak-to-peak signal voltages resulting form inputs A and B of the dual input oscilloscope were measured and compared. The parallel tapping of the outputs (inputs A and B of the oscilloscope) favours the comparison of the outputs with input B acting as a bypass. This also allows us evaluate the effectiveness of the improvised sections of the circuit especially as it approaches resonance.

Results obtained

By monitoring the response of the circuit on the oscilloscope to the signal fed in from the signal generator at given frequencies simultaneously, the following data was obtained. This was used to plot a graph from which the resonance frequency (f_r) was determined. This experiment was carried out at a temperature of about 28°C. The paper used as the dielectric has a thickness (d) of 9.7×10^{-4}m.

FREQ (MHz)	INPUT SIGNAL (mV)	OUTPUT SIGNAL (mV)	Amplitude (mm)
3.33	220.00	64.00	29.00
3.85	210.00	80.00	38.00
5.00	190.00	84.00	44.00
5.56	190.00	100.00	53.00
6.58	180.00	112.00	62.00
7.81	175.00	94.00	54.00
8.33	240.00	84.00	35.00

Table 1 Data from oscilloscope.

The graph of amplitude (A) was plotted against frequency (MHZ), from which a resonance curve was obtained. The resonance frequency f_r was determined as 6.58 MHZ.

Determination of dielectric constant

At resonance frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad (14)$$

From

$$C = \frac{1}{\omega^2 L} = \frac{\varepsilon_p \varepsilon_o A}{d} \quad (15)$$

where ω is the angular frequency (Radian/Secs) = $2\pi f_r$, ε_p the relative permittivity of paper, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/M, A the area of the capacitor plate (m^2) and d the thickness of the paper, equation (15) can be rewritten as:

$$\varepsilon_p = \frac{d}{\omega^2 L \varepsilon_o A} = \frac{d}{4\pi^2 f_r^2 L \varepsilon_o A} \quad (16)$$

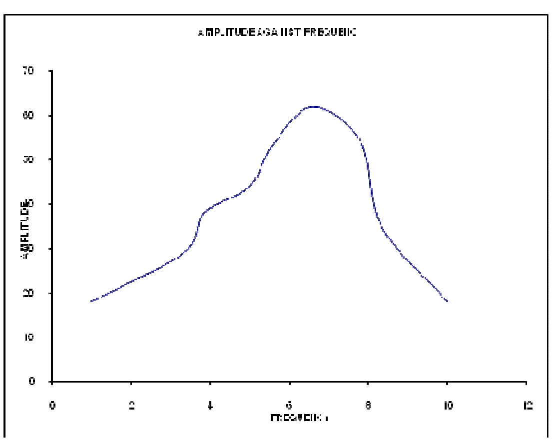


Fig 7 Resonance curve of the A4 paper

Table 2 Reference values of Dielectric Constants of some Industrial Products (Hamon, 2000; Elwell et al., 1978)

Materials	Dielectric Constants	Materials	Dielectric Constants
Air	1.0	Oil saturated paper	4.0
Cellulose	3.2 - 7.5	Water	48 – 88
Glass	3.1 – 10	Sugar	3.0
Transformer Oil	2.2 - 2.4	Silicon dioxide	4.5
Paper	1.6 - 4.5		

L was computed to have a value of 2.64×10^{-4}H. The area "A" of the capacitor plate was obtained as $4.9 \times 10^{-4} m^2$ (where $r = 12.5 \times 10^{-3}$m). The resonant frequency was computed as 6.58×10^6 HZ, while ε_o and ε_r were obtained as 8.854×10^{-12} Fm^{-1} and 1.203 respectively. Thus, the dielectric constant of paper was obtained to be 1.203, which is close to the expected value. Table 2 shows the results for the dielectric constants of some common industrial materials, which are used in various spheres under human endeavour.

Discussion and conclusion

Capacitance is a parameter whose value relies heavily on the presence of charges – whether stray or induced. These charges, when present in a substance, influence its dielectric behaviour. The theory of dielectrics predict that dielectric behaviour relies heavily on properties such as polarisation, conductivity, energy storage and power factor, loss tangent and Q factor and most importantly the relative permittivity of a substance. These properties were analysed in course of this research. The aim of adopting a modified version of the Hartshorn and Ward technique is primarily to eliminate the effects of stray capacitances, resistance and inductances that would have been the problem in using other methods such as the Schering method. Also, this method sidelines the complex numerical integrations that would have been required using the step response technique. More importantly is the adaptation to a useful frequency band – the audio frequency, which makes this method a natural tool for this research. This method is based on the use of precision electrodes with minimal leads in which resonance is attained by adjusting the variable capacitor. The half power bandwidths b_o and b_i of the resonance curve are found by fine tuning the capacitor with air and then with dielectric inserted appropriately. The capacitance of the specimen is a function of the change in variable capacitance needed to achieve resonance again with dielectric inserted. The relative permittivity of the A4 paper, ε_p, was found from using equation (16). The dielectric constant obtained for A4 paper is consistent with standard results presented in Table 2. The effects of stray capacitance emanating from the connecting wires, stray series capacitance, which reduced effective capacitance of the circuit and the effect of the parallel stray inductance which reduced effective inductance, only resulted in minor discrepancies when compared with reference values of similar materials as presented in Table 2. It was also noticed that the factor by which the capacitance increases depends on the nature of the dielectric more than it does on the size and shape of the capacitor. The relative permittivity varies with temperature, pressure and frequency. The capacitance of a capacitor is several times larger when filled with dielectric than in vacuum, which implies that materials with very high relative permittivity provide better capacitance than those with less. The experimented circuit was mounted on bread board for ease of experimentation.

The resonance method of determining the dielectric constant of specimens offers a reliable method that may be applied to several specimens for various applications. For instance, some capacitors impregnated with paper as their dielectric material. Considering the role played by the permittivity of paper used as dielectric material, this research offers an avenue of using alternative dielectric material in the design of various capacitor types depending on the value of capacitance required. This same method may be used in determining the relative permittivity of industrial materials such as lubricating oils, transformer oils etc.

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